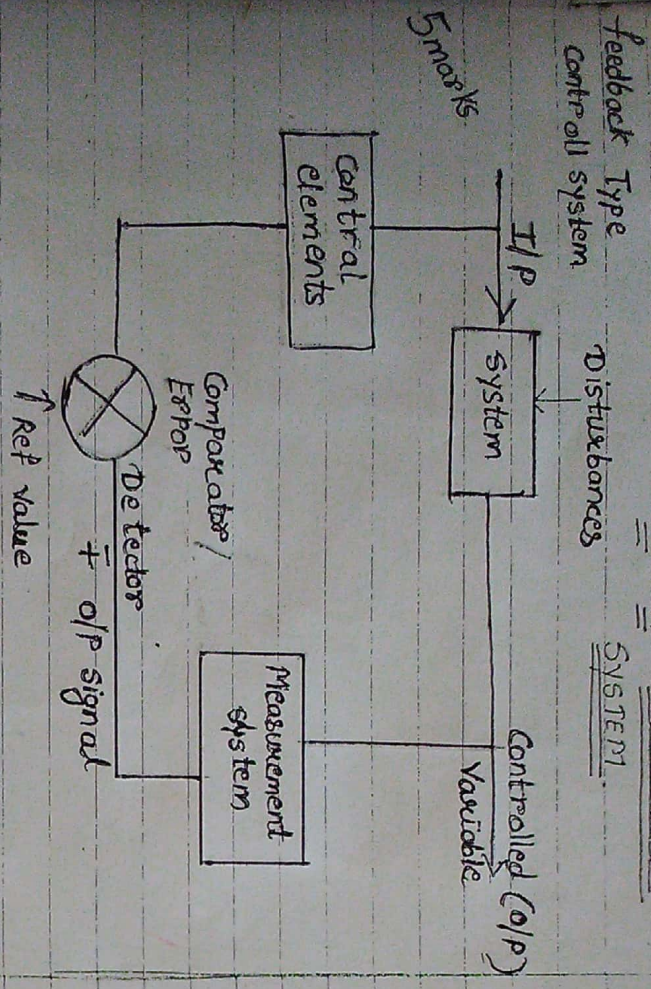


# UNIT I → Kirti Sir

## BASIC FUNCTIONAL ELEMENT OF A MEASUREMENT SYSTEM



(5.E) 1. Transducer element that senses & converts the desired value / output into the more convenient & predictable form to be handled by the system.

- Signal conditioner (or) intermediate modifying element for processing the output of a transducer in a suitable form.
- Data presentation element (DPE) for giving the information of the measured variable in a quantitative form.

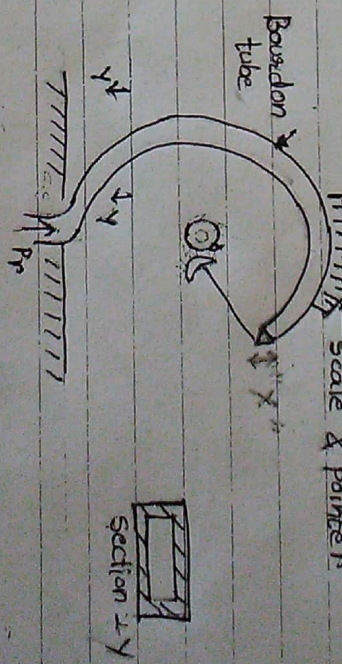
2. Identify the various functional element of

a measurement system given below gives measurement devices, pressure m.d, liquid level m.d, force m.d, temperature m.d etc)

4. Auxiliary elements (calibration elements, build in calibration facility)

b) External power elements → To facilitate the working of one or more of the elements, like the transducer elements, signal conditioning element, data presentation element, feedback element.

### Bourdon Tube Pressure Gauge



1. The pressure applied to the hollow oval shaped bend tube known as Bourdon gauge (C shape) deforms the cross section of the tube & causes the relative motion, relative to the applied pressure, of the free end of the tube with respect to its fixed end



2. Tube acts as a transducer as it convert the desired input i.e. pressure into a displacement at its free end.
3. Its displacement is amplified by the combine lever & gearing arrangement which may be referred as the signal conditioning element.
4. The movement of the pointer attached to the gear on a scale gives an indication of the pressure & does the pointer & the scale constitute data presentation element.

Transducer element:

It senses the desired input in one physical form & convert it to output in another physical form. For eg. the input variable to a transducer could be pressure, acceleration & temperature & output may be of the transducer may be displacement, voltage, resistance change depending on the type of the transducer element. (If the units of the input & output signal are same then it is termed as transformer.)

1) Venturimeter, thermocouple, micrometer, liquid in glass thermometer, LVDT (liquid volume distribution transformer)  
 Linear Linear variable differential transformer

Sr.no	Type of device	Input	O/P	Working principle	Type of device
1.	Temp to the transducer	Temp to the transducer			

2. Temp to the thermometer  
 2. Thermometer  
 Temp to the thermometer covered by the mercury  
 level rise distance covered by the mercury  
 Because of the capillary rise in liquid level



no	Type of device	ITP to the device	QPF to the device	working principle	range & appl <sup>n</sup>
1)	Thermocouple	Temp to the transducer	voltage	An emf is generated across the junction of two dissimilar metals when the junction is heated	
2)	Thermometer	Temp to the thermometer	level rise	Because of the expansion of the capillary covered by the mercury (the mercury) is the rise in liquid level.	

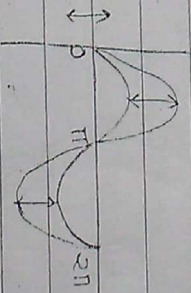
### Characteristics of transducers:

- 1) It should recognize & sense the desired input signal.
- 2) It should have good accuracy.
- 3) It should have good precision.
- 4) It should have good linearity.
- 5) It should be easily available & reasonably price.
- 6) easily portable.

### • Signal Conditioning element:

The output of the transducer element is usually too small to operate an indicator or a recorder, so it is suitably processed & modified. In the signal conditioning element so as to obtain the output in the desired form.

(a) Amplification: Amplification means increasing the amplitude of the signal without affecting its wave form. The reverse phenomenon is termed as 'attenuation' (decreasing the amplitude or reduction of amplitude signal).



1. Mechanical amplifying elements (levers)
2. Optical amplifying " (lenses)
3. Electrical " " (mikes)

Signal filtration: It means the removal of unwanted voice signals.

Data presentation elements (DPE): Read or seen by the experimenter



## Characteristics of DPE:

1. As fast response as possible.
2. have very small inertia, friction effects.

## Classification of Instruments:

1. Null type instruments. (Weight balance m/c)
2. Reflection type instruments.  
(Eg. Bourdon tube pressure gauge.)

3. Automated type instruments

4. Manually operated instruments.

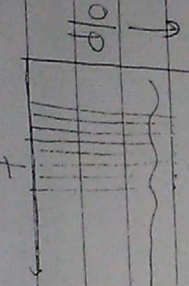
Any instruments which requires the services of human operator is a manual type of instrument.

It becomes automatic if the manual operation is replaced by an auxiliary device incorporated in the instrument.

## Analog & digital type of instrument:

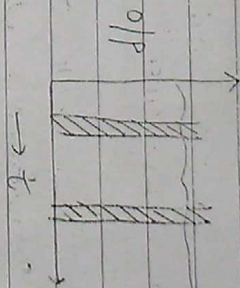
An analog

is that present the physical variable of interest in the form of continuous variations with respect time



Digital type of instrument: That present the physical variable represented by digital quantities which are discrete & vary in steps.

Further each digital no. is a fixed sum of equal steps which is define by that number. Graphical representation is as given below



\* SELF-GENERATED AND POWER OPERATED TYPE INSTRUMENTS:

Self generated: The energy requirement of an instrument met entirely from the input signal for eg. an exposed meter of a camera which is in effect, a photo voltaic cell (is also a self generating instrument) Eg. Bourdon tube pressure gauge

Working Principle: In this the incident light energy whose intensity is being measured is supplied the entire energy for generating the proportional amount of output voltage in semiconductor junction.



Power Operated instruments:

The instruments which requires the external source of power to operate then they come under power operated type of instrument. For eg, Compressor, Electric Iron.

\* Standards of measurement:

1. It is defined as the physical representation of the unit of measurement.

a. Primary std: are the devices maintained by standard organizations, national laboratories in different parts of the world.

b. Secondary std: are the basic reference standards employed by industrial measurement laboratories.

c. Tertiary or working standard of meas: These are accurate devices which are commercially available & duly checked & are certified against the primary as well as secondary standards.

\* Calibrations: It is the act or result of ~~the~~ quantitative comparison between a known standard and

the output of the measuring system measuring the same quantity. For eg. Thermometer

Static Characteristic: In a number of situation, the desired input to the instrument may be constant or varying slowly with respect to time. For eg accuracy, precision, resolution, sensitivity, linearity, hysteresis, drift, overload capacity, impedance loading are usually good enough to give quantitative description of an instrument.

(We also come across certain situation where the desired input is not constant but varies rapidly with the time, in such cases the dynamic characteristic of the instrument should also be known.)

In general the overall quantitative performance, qualities of the instrument/gauge are represented by both their static & dynamic characteristics. For time independent signals only the static characteristics need to be considered.

Static Performance Parameters / static char:-

1) Accuracy: It is defined as the closeness of the instrument output to the



True value of the measured quantity (As per standards). It is given as percentage of true value.

$$\% \text{ of True Value} = \frac{\text{Measured value} - \text{True value}}{\text{True value}} \times 100$$

2. Precision: A highly precise instrument is one that gives the same output repeated information for a given input information, when the reading is repeated a large no. of times.

For eg, Bourdon tube pr. gauge, thermometer.

3. Threshold: It is defined as the minimum value of input, below which no output can be detected.

It is not zero because of various factors, like friction between moving parts, play or looseness in the joints, inertia of the moving parts etc.

Input to Bourdon tube pr. gauge (bar)      Output to the Bourdon tube pr. gauge (mm)

0.5      0



4. Resolution: If the input is increased slowly from some arbitrary non-zero input value again the output does not change at all until a certain input increment is exceeded.

This increment is called the resolution. Thus it is defined as the input increment that gives some small, but definite numerical change in the output.

Thus, resolution defines the smallest measurable input change while threshold defines the smallest measurable input. Eg thermometer, Bourdon tube pr. gauge, venturimeter, galvanometer, Thermocouple.

5. Static Sensitivity [Scale factor / Gain]:

It is defined as the magnitude of response (output signal) to the magnitude of the quantity being measured (input signal). Mathematically it is given as

$$K = \frac{\text{Change of output signal}}{\text{Change of I/P signal}} = \frac{\Delta Q_o}{\Delta Q_i}$$

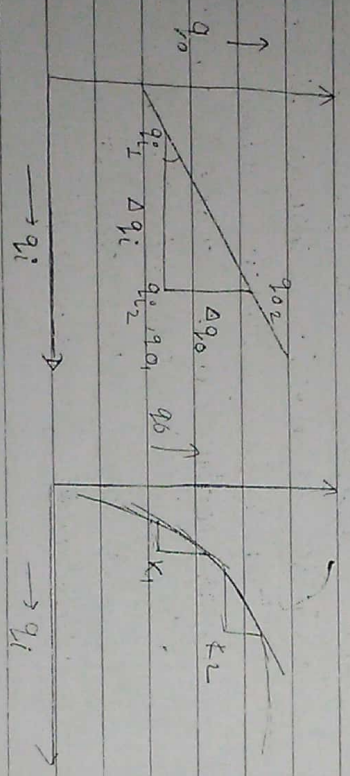
So,  $Q_i =$  values of output, input signal



It is also represented by, the slope of the input/output curve, if the ordinates are represented in actual units.

With a linear calibration curve, the sensitivity is constant, if the relationship between the I/P, O/P is not linear, the sensitivity varies with the I/P value, and is defined as

$$K = \frac{\Delta q_o}{\Delta q_i}$$

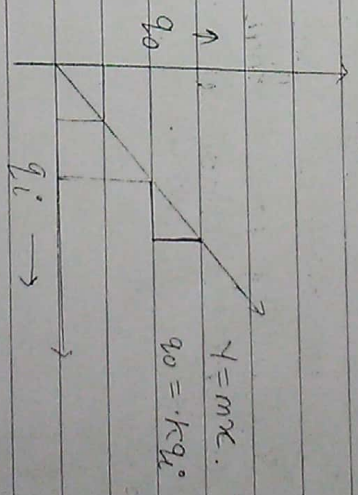


The units of responses  $q_i, q_o$  depends on the different properties (Pa, F, T, Q) eg, sensitivity of a strain gauge extension is directly proportional to the applied force & can be given as 450  $\mu\text{V}/\text{mm}$ . Also, the sensitivity of a thermocouple [Cu-Constantan] is 60  $\mu\text{V}/^\circ\text{C}$ .

Note: It may be noted that in certain operations, the appearance of the

sensitivity, is commonly used. It is termed as Sensitivity factor or the deflection factor.

Linearity: Manufacturers of instruments always attempt to design their instrument so that the output is a linear function of the I/P. If the deviations from the ideal are called as linearity tolerances.



2. % the deviation of the O/P of the instrument from the best fitting straight line does not vary with the I/P line then it is a case of non-linear.

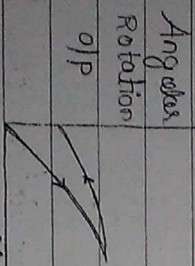
3. The deviations of the O/P of the instrument from the idealize straight line, vary with the I/P



4. Combine independent & proportional to the I/P (in certain case the deviations of the o/p may not vary with the I/P for a part of the range & may show proportional variation for the rest of the range)

\* Static Characteristics Hysteresis:

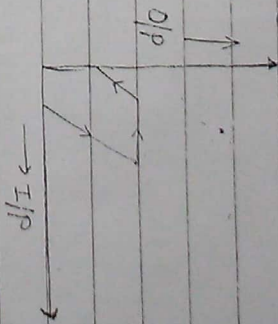
(i) Consider a situation in which the pressure gauge as the output pressure slowly & smoothly varied from zero to full scale & then back to zero. If these where no friction due to sliding of moving parts than input & output graph might a pair as below.



(ii) The ~~less~~ non-coincidence of loading (increasing I/P) & unloading (decreasing I/P) curves is due to the internal friction or hysteresis damping of the stressed parts (mainly the spring). All the energy put in the stressed parts upon loading is not recoverable during unloading because

of the second law of thermodynamic which indicates that no process is perfectly reversible in the world.

(iii) If internal friction is zero, but external sliding friction is there, the result might be

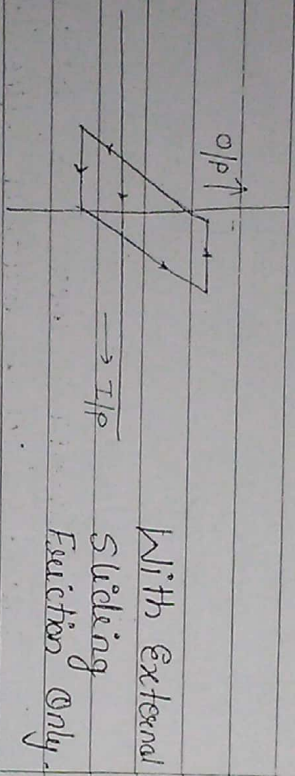
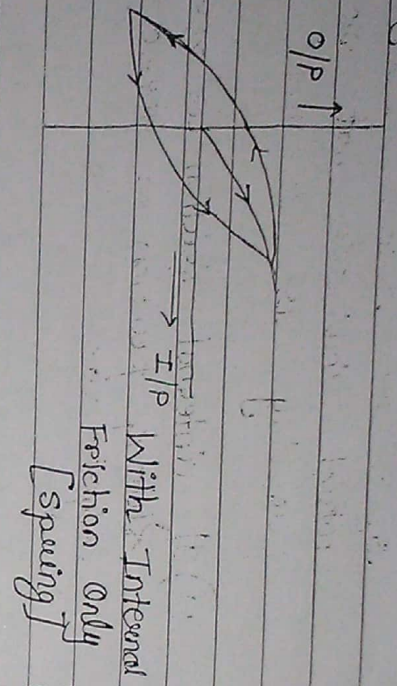


Whenever there is solid contact between surfaces, friction (due to coulomb's friction) comes into play. It is defined as the force or torque necessary to initiate the motion of the sliding & moving parts. If there is any free play or clearances in the mechanism of an instrument a curve of similar shape will result.

(iv) For instruments with a useable range on both sides of zero, the o/p, I/P characteristic of the instrument takes the shape of a hysteresis curve known as hysteresis loop, as given below.



Unit a note: O/P  
 Input & draw  
 I/P, O/P dir.



Note: In a given instrument a no. of causes other than just mentioned may combine to give overall hysteresis effect.

Backlash :-

It is defined as the max. distance or angle through which any part of the mechanical system may be moved in one direction without causing motion of the mating parts. The I/P, O/P characteristic is similar to hysteresis loop. Backlash can be minimized if the components are made to very close tolerances.

Drift :-

It is an undesired gradual departure of O/P over a period of time. It is caused by wear, tear, high stresses developing at some parts & contamination of primary sensing element.

Range & span :- The region betw the limits

within which an instrument is designed to operate for measuring is called the range of the instrument.

The range is expressed by indicating lower & upper value.

Span indicates the algebraic diff betw the upper & lower value. For eg: In a thermometer if the range is given as  $-10^{\circ}\text{C}$  to  $+80^{\circ}\text{C}$  then the span for the same is  $80 - (-10^{\circ}\text{C}) = 90^{\circ}\text{C}$

Error :-

It is defined as the ratio of difference betw the measured actual value & the true value to the true value

$$= \frac{M.V - T.V}{T.V} \times 100$$



\* Types of error :- [On the basis of Numerical value]

1. Systematic or Cumulative Error :- If the errors have same magnitude & sign, they are called systematic errors. As the algebraic signs are same, they tend to accumulate & hence are known as cumulative error.

2. Accidental or Random Error :- This can have +ve or -ve sign & magnitude is not always same. This error are in either direction, they tend to compensate one another. So they are also called as compensating error.

3) Gross / Total Error :- These are certain errors which can not be classified as either systematic or random as they are partly systematic & partly random. Such errors are called gross errors.

\* Other types of Error :-

1) Instrument Error :- They are many factors in the design & construction of instrument that limit the accuracy

attainable.

Instrument possess some inherent accuracy & certain additional inaccuracies develop with use & time.

Environmental errors :- The instrument locations & the environmental error are introduced by using an instrument in conditions, different for which it has been designed assembled & calibrated. The diff condn are because of the temp fluctuations, humidity factor, altitude etc.

For eg :- If mercury in glass thermometer is located at a particular zone where some intensities is high then the metallic bulb of the thermometer will be shined very much & it is harmful for the designed of the thermometer.

3) Translation & Signal Transmission Error :- It includes. The non capability of the instrument to follow the input changed in the measured quantity due to inertia & hysteresis effect.

It may also be resultant from unwanted disturbances such as noise at the higher decibels.



Observation error: Even when instrument is properly selected, care fully installed and properly calibrated, slight errors in the measurement occurs due to certain failure of some part of the observer.

i) Parallax (i.e. apparent displacement when the line of vision is not normal to the scale).

ii) Personal Bias (i.e. tendency to read very high value, very low value or the reading is seen very fast)

3) Operational error: A different type of error meter will read inaccurately if it is placed immediately after a valve or a bend.

READABILITY AND LEAST COUNT: The term readability indicates the closeness <sup>of</sup> which the scale of instruments may be read.

The term least count signifies the smallest difference that can be detected on the instrument scale.

Stiction (static friction) is force or torque that is necessary to indicate motion from rest.

Problem on best Fit Line:

1) For the following observations on a test data, obtain the equation of best fit line, linearity of the sensitivity of the system. (Assume the best fit line as  $y = mx + c$ ).

N	LOAD	OP
10	16	0.8
22	33	1.7
33	38	2.7
38	43	3.8
43		4.7
		6.7

Sol<sup>n</sup> For the above situation, the governing eqn is  $y_0 = mx + c$

For analysing the problem, we compare the table as

ser.no	LOAD (N)	(mV) O/P
1	10	0.8
2	16	1.7
3	22	2.7
4	33	3.8
5	38	4.7
6	43	6.7

$m = 6$        $\sum y_i = 162$        $\sum x_i = 204$

full scale measurement



$q_i^2$	$q_i q_0$	$q_0 C$	$q_0 - q_0 C$
100	8	0.673	0.127
256	27.2	1.683	0.067
484	59.4	2.593	0.107
1089	125.4	4.353	-0.553
1444	178.6	5.153	-0.153
1849	288.1	5.953	-0.747

$\sum q_i^2 = 5222$   
 $\sum q_i q_0 = 686.7$   
 $\sum q_0 C = 28.7$   
 $\sum (q_0 - q_0 C) = 0.747$  (max deviation)

$$a = \frac{\sum q_0 \sum q_i^2 - \sum q_i q_0 \sum q_i}{n \sum q_i^2 - (\sum q_i)^2}$$

$$a = \frac{(20.4 \times 5222) - (686.7 \times 162)}{(2 \times 5222) - 162^2}$$

$$a = -0.927$$

$$m = \frac{n \sum q_0 q_i - \sum q_0 \sum q_i}{n \sum q_i^2 - (\sum q_i)^2}$$

$$q_0 = m q_i + a$$

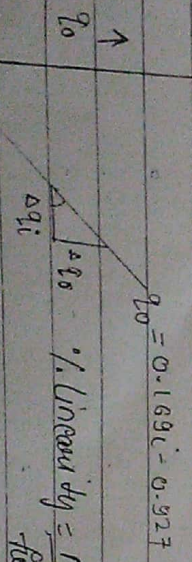
$$q_0 = 0.16 \times q_i + (-0.927)$$

$$q_0 = 0.16 \times 22.4 - 0.927$$

$$q_0 = 2.593$$

Sensitivity =  $m = 0.16$

Also,  $q_0 = 0.16 q_i - 0.927$



$$\Delta q_i = 10$$

$$\Delta q_0 = 0.747$$

$$\% \text{ Linearity} = \frac{\Delta q_0}{\Delta q_i} \times 100$$

$$= \frac{0.747}{10} \times 100$$

$$= 7.47\%$$

### Dynamic characteristic

When the instruments are required to measure an I/P which a varying with time, the dynamic behavior is equally important. The signals cannot be impressed instantly & the mass of the capacitances (thermal, electrical or fluid), causes slowness in the measurement system. The system does not settle to its equilibrium steady state immediately after the application of the input signal.

Following are the some of the important dynamic characteristic which is very

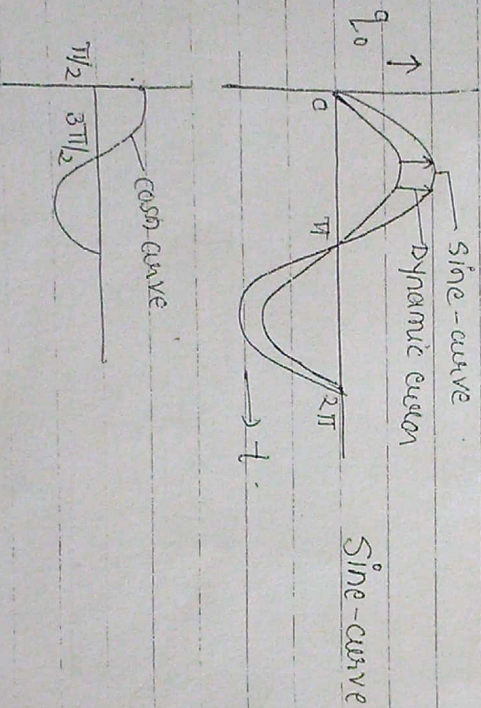
1) Speed of Response: It is define as the response to a change in the value of the quantity being measured.

2) Measuring lag: It refers to delay in the response to a change in the input signal, This delay is caused by capacitance, inertia or resistance of measurement.

3) Fidelity & Dynamic error: Fidelity is define as the degree of closeness



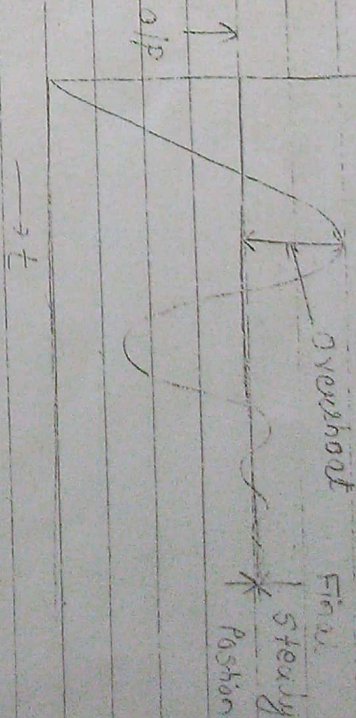
with which the system records the signal, it refers to the ability of the system to reproduce the input in the same form as that of the input, if the I/P is sine wave the output should also be of sine wave. The difference betn the indicated value & the true value of the time varying quantity is the dynamic error,  $\therefore$  static error is assume to be zero.



#### ④ Overshoot :-

Because of mass & inertia, the pointer does not immediately come to rest. The pointer goes beyond the steady state, if it overshoot.

The overshoot is defined as the maximum amount by which the pointer moves beyond the steady state.



#### ⑤ Dead Time & Dead Zone :-

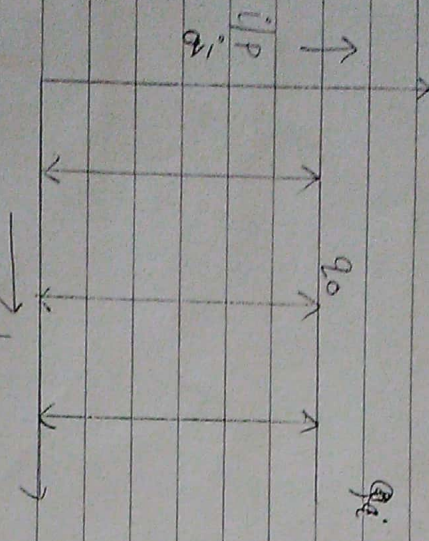
Dead Time is define as the time required to begin to respond to change in the measured quantity. The dead zone defines the largest change in the measured quantity to which the instrument does not respond, it is cause due to friction, backlash or hysteresis in the instrument. The graphical representation is as given below.



Standard Test Inputs:

The dynamic behaviour can be determined by applying some known & predetermined I/P signals & then studying the behaviour of o/p signals.

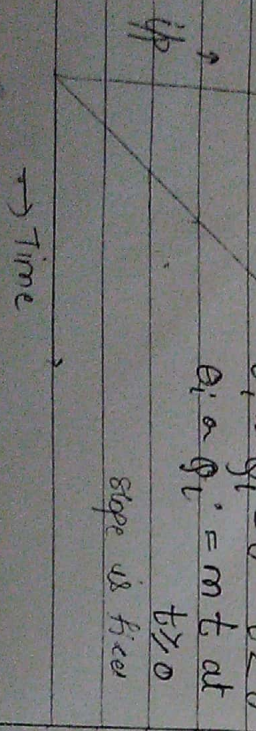
1) Step Input:



where  $q_0$  is a constant value of the I/P signal "q\_i"

$q_i = 0$  at  $t < 0$   
 $q_i = q_0$  at  $t \geq 0$

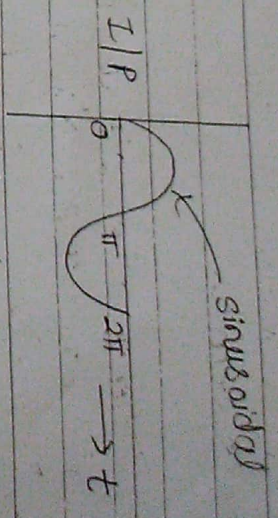
2) Ramp or Linear I/P:



$q_i = 0$  at  $t < 0$   
 $q_i = m t$  at  $t \geq 0$

Slope is fixed

3) Sinusoidal or sine wave I/P:



The I/P is a cyclic variation, it varies sinusoidally with a constant maximum amplitude & the eqn for the same is given as

$$q_i = A \sin \omega t$$

where A - Amplitude  
 $\omega$  - Frequency in rad/sec

A general measurement system can be mathematically describe by the differential eqn as given below,

$$[A_n D^n + A_{n-1} D^{n-1} + \dots + A_1 D + A_0] \theta_i = [B_m D^m + B_{m-1} D^{m-1} + \dots + B_1 D + B_0] \theta_0$$

Zero Order System:

All the A's & B's other than  $A_0$  &  $B_0$  are assumed to be zero in the general eqn. The eqn takes the form

$$A_0 \theta_i = B_0 \theta_0$$

By further reduction

$$\theta_i = \frac{B_0}{A_0} \theta_0$$



where  $K$  is known as static sensitivity of the system.

Comments: Hence, the o/p is directly proportional to i/p, No matter how the i/p varies. The o/p is a faithful reproduction of i/p without any distortion or time lag. Some eqs of zero order system are mechanical levers, amplifiers & linear electrical potentiometer.

First Order System: All the A's & B's other than  $A_0, B_0$  &  $A_1$  are taken as zero in the general mathematical differential eq<sup>n</sup>. Now, the eq<sup>n</sup> becomes

$$A_1 \frac{d\theta_0}{dt} + A_0 \theta_0 = B_0 \theta_i$$

$$[A_1 D + A_0] \theta_0 = B_0 \theta_i$$

$$\frac{A_1}{A_0} \frac{d\theta_0}{dt} + \theta_0 = \frac{B_0}{A_0} \theta_i$$

$$T = \frac{d\theta_0}{dt} + \theta_0 = K \theta_i$$

where  $T$  is the time constant & is given as,

$$\frac{A_1}{A_0}$$

&  $K$  is the static sensitivity of the system which is given by  $\frac{B_0}{A_0}$

$$[TD + 1] \theta_0 = K \theta_i$$

examples of this system are mercury in glass thermometer, thermistors, thermocouples, velocity of a free falling mass, resistance capacitance network

$$A_1 \frac{d\theta_0}{dt} + A_0 \theta_0 = B_0 \theta_i$$

Assume, the static sensitivity  $K=1$  for studying the dynamic behaviour. For solving the above eq<sup>n</sup> the eq<sup>n</sup> can be written as

$$(1 + TD) \theta_0 = \theta_i$$

Transient Response: It is obtained from the sol<sup>n</sup> of the auxiliary eq<sup>n</sup> (complementary func<sup>n</sup>) and is given as



$$(1 + \tau D) \theta_0 = 0$$

$$\left(D + \frac{1}{\tau}\right) \theta_0 = 0$$

$$y = C_1 e^{m_1 x} + C_2 e^{m_2 x} \dots \dots y = 0$$

$$\left[D - \left(-\frac{1}{\tau}\right)\right] \theta_0 = 0$$

$$\theta_0 = C e^{-t/\tau}$$

$$\theta_0 = C e^{-t/\tau}$$

The transient response would be same for different standard I/P.

(b) Steady state response  $\dot{\theta}$  (particular integral)

$$(1 + \tau D) \theta_0 = \theta_i$$

$$\theta_0 = [1 + \tau D]^{-1} \theta_i$$

$$\text{as } (1 + x)^{-1} = 1 - x + x^2 - x^3 + \dots$$

$$(1 + \tau D)^{-1} = 1 - \tau D + \tau^2 D^2 - \tau D^3 + \dots$$

$$= [1 - \tau D + \text{terms in } D^2 \text{ \& higher}] \theta_i$$

(1) For step input.

$$\theta_0 = \theta_i$$

$$\theta_0 = C e^{-t/\tau} + \theta_i$$

The constant C can be determined from the initial conditions,

$$\text{as } \theta_0 = 0 \text{ at } t = 0$$

$$\theta_0 = C e^0 + \theta_i$$

$$\theta_0 = C + \theta_i$$

$$C = -\theta_i$$

$$\theta_0 = \theta_i (1 - e^{-t/\tau})$$

gn the non dimensional form it is given as  $\theta_0 = \theta_i (1 - e^{-t/\tau})$

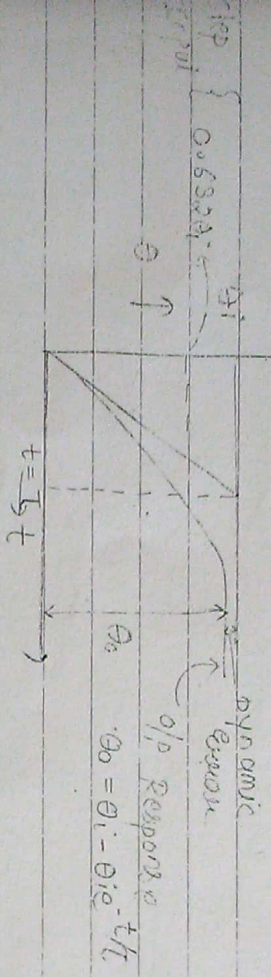
$$\frac{\theta_0}{\theta_i} = (1 - e^{-t/\tau})$$

Analysis  $\dot{\theta}$  The final eq<sup>n</sup> can be written as  $\theta_0 = \theta_i (1 - e^{-t/\tau})$

1) The numerical value of transient response drops with time & after sufficiently long time the value approximates to zero & thus when the time is very large  $\theta_0 = \theta_i$  and the transient response becomes the dynamic error.



2. The time constant of  $\theta_i \tau$  indicates the speed of response i.e. how fast the system reached the steady state value. A large time constant corresponds to a slow system response & vice versa. Then, in first order system attempts to be made to minimize  $\tau$ , for accurate dynamic measurement.



3. At  $t = \tau$ ,  $\frac{O_o}{O_i} = 0.632$ ,

thus the time constant  $\tau$  is defined as the time to reach 63.2% of a static or steady state value.

4. The residual difference between the I/P & the response curve is called as the dynamic error. Therefore, the dynamic error will be given as

$$\begin{aligned} \text{Dynamic error} &= O_i - O_o \\ &= O_i - (O_i - O_i e^{-t/\tau}) \\ &= O_i - O_i (1 - e^{-t/\tau}) \end{aligned}$$

$$\text{Dynamic error} = O_i (1 - e^{-t/\tau})$$

5. The speed of response of a system is defined in terms of settling time which represent the time taken for the system to reach to within a certain % of tolerance band of the final steady state value.

A small settling time is the indication of fast response. Typical would be 2%, a 5% settling time.

It is often assumed that the instrument reaches its final steady state value after an interval of time.

$$t = 5\tau$$

Dynamic error can also be given as;

$$e_m = O_i e^{-t/\tau}$$

In dimensionless form  $\frac{e_m}{O_i} = e^{-t/\tau}$



$t/T$	0	1	2	3	4	5	$\infty$
$\frac{\theta_0}{\theta_i}$	0	0.632	0.865	0.95	0.982	0.993	1
$\frac{e_m}{\theta_i}$	1	0.368	0.135	0.0498	0.0183	0.0067	0

② Ramp Input

Let  $\theta_i = mx$ , so the governing differential eq<sup>n</sup> is of the form

$$(1 + \tau D) \theta_0 = mx$$

The complementary func<sup>n</sup> is given by  $\theta_0 = Ae^{-t/\tau}$

A particular integral will be  $\theta_0 = (1 + \tau D)^{-1} mx$

$$= [1 - \tau D + \tau^2 D^2 - \dots] mx$$

$$= mx - \tau \frac{d}{dt} (mx) + \dots$$

$$= mx - m\tau$$

The complete response will be given as

$$\theta_0 = mx - m\tau + Ae^{-t/\tau}$$

Now from the boundary cond<sup>n</sup>  $\theta_0 = 0$  at  $t=0$

$$0 = 0 - m\tau + A$$

$$A = m\tau$$

$$\theta_0 = mx - m\tau + m\tau e^{-t/\tau}$$

④ For a different value of time constant a single value of  $m$  tabulate the

The dynamic response is as now

$$\theta_0 = m [t - \tau (1 - e^{-t/\tau})]$$

The dynamic error is given as,

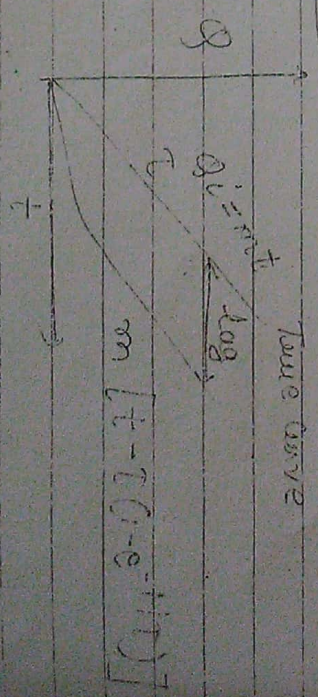
$$e_m = \frac{\theta_i - \theta_0}{\theta_i}$$

$$e_m = m\tau - m\tau e^{-t/\tau}$$

$m\tau \rightarrow$  Steady state response

A  $m\tau e^{-t/\tau}$  transient response.

$$\frac{e_m}{m\tau} = (1 - e^{-t/\tau})$$



① The term  $m\tau e^{-t/\tau}$  gradually decreases with time and hence is called the transient



The term (1st term) independent of time will continue to exist & is called as steady state ( $t \rightarrow \infty$ )

2) The transient error decreases rapidly if  $T$  is made small. Again it indicates that the system attains the steady state error is directly proportional to  $mT$  & therefore the larger the time constant larger the magnitude of the error.

3) The output response curve always lags behind the I/P curve by a constant amount known as measuring length.

4) For different value of time constant & a single value of  $m$  tabulate the relationship for  $\theta_r$  &  $\theta_o$  (uses test table for step I/P of constant magnitude)



(iii) **Zonal Drift.** In case the drift occurs only over a portion of span of an instrument, it is called zonal drift.

There are many environmental factors which cause drift. They may be stray electric and magnetic fields, thermal emfs, changes in temperature, mechanical vibrations, wear and tear, and high mechanical stresses developed in some parts of the instruments and systems.

**Drift** is an undesirable quantity in industrial instruments because it is rarely apparent and cannot be easily compensated for. Thus it must be carefully guarded against by continuous prevention, inspection and maintenance. For example, stray electrostatic and electromagnetic fields can be prevented from affecting the measurements by proper shielding. Effect of mechanical vibrations can be minimized by having proper mountings. Temperature changes during the measurement process should be preferably avoided or otherwise be properly compensated for.

**Example 2.6.** A dial gauge is used to measure pressure in a vessel. The pivot is not exactly at the centre and as a result of which the readings are subject to a systematic error. It was found that the imperfection makes the readings too large in a linear fashion. The readings are 6.895 kN/m<sup>2</sup> for a dial reading of zero and 27.58 kN/m<sup>2</sup> for a reading of 150. What would be the value of pressure for a dial reading of 100 ?

**Solution.** This is a case of zero drift. The dial reading is offset by 6.895 kN/m<sup>2</sup> at zero pressure and reading at 150 indication of dial is 27.58 kN/m<sup>2</sup>. Since the variation is linear, the indication at dial reading of 100 is :

$$\frac{27.58 - 6.895}{150} \times 100 + 6.895 = 20.685 \text{ kN/m}^2.$$

### 2.13. NOISE

A spurious current or voltage extraneous to the current or voltage of interest in an electrical or electronic circuit is called **Noise**. In fact, noise is a signal that does not convey any useful information. Noise may be generated external to a particular system of interest and enter the system in various ways or it may be generated inside the system of interest.

The effect of noise in a measurement system may be an annoying "hum" in the speaker of a radio receiver owing to 50 Hz power line frequency noise, to the transmission of incorrect data in telemetering systems or data communication systems, to life threatening situations caused by incorrect interpretation of waveforms related to vital human organs in biomedical instrumentation systems where the expected (desired) signals are intrinsically weak. This is because the extraneous noise signals constitute a background against which the desired signal may be read.

**2.13.1. Signal to Noise Ratio (S/N).** Noise is an unwanted signal superimposed upon the signal of interest thereby causing a deviation of the output from its expected value. The expected value of the output is the one that is obtained only when desired signal or signal of interest is present. The extent to which noise becomes important in the measurement systems depends upon the relative value of magnitude of unwanted signal (noise) to that of signal of interest. If the magnitude of unwanted signal (noise) is small as compared with that of signal of interest, then signal to noise ratio (S/N) is large and therefore the noise becomes unimportant.

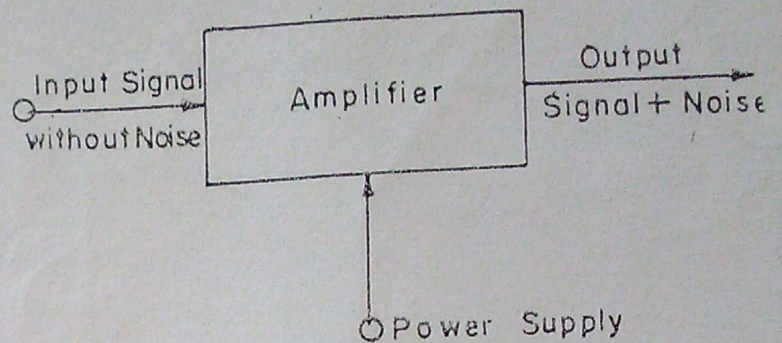
The ratio of desired signal to the unwanted noise is called **signal to noise ratio** and is expressed as :

$$\frac{S}{N} = \frac{\text{signal power}}{\text{noise power}} = \frac{(\text{signal of interest expressed in volt})^2}{(\text{unwanted noise expressed in volt})^2} \quad \dots(2.10)$$



In any measurement system, it is desirable to have a large signal-to-noise ratio. This can be achieved by increasing the signal level without increasing the noise level or decreasing the noise level with some suitable technique.

**2.13.2. Sources of Noise.** The noise in the output is broadly of three types : (i) generated noise, (ii) conducted noise, (iii) radiated noise. This can be illustrated by considering the case of an amplifier which is commonly used in measurement systems to identify the different sources of noise. The block diagram of the amplifier is shown in Fig. 2.3.



(i) **Generated Noise.** Suppose the

input signal contains no noise. The power supply serves as a source of energy for the operation of amplifier. The output signal is amplifier gain times the input signal plus a noise signal. One of the possible sources of noise is on account of internal components of the amplifier like resistors, capacitors and transistors etc. Therefore the noise in this case is generated inside the amplifier and therefore is called **Generated noise**. The internally generated noise is on account of components like resistors, capacitors, transistors etc. as stated above.

The conductive portion of a resistor consists of a regularly arranged groups of atoms that maintain the same general physical position in the conductor. These atoms contribute conduction electrons due to which a current flows. Although the atoms maintain their general physical position, they are in a state of rapid vibratory motion on account of temperature and thermal effects. This vibratory motion of atoms is transferred to the conduction electrons, thereby producing a noise component of current. Since this noise is temperature dependent, it increases with internal heating ( $I^2R$  loss) or with an increase in the ambient temperature. This noise is called **Johnson noise**.

The vibrations produced by thermal effects within a resistor cover a wide frequency range, and therefore the noise generated consists of a wide spectrum of frequencies. This wideband noise is sometimes called **White noise**.

The internally generated noise in resistors can be reduced by lowering the internal temperature. Special film and glass substrates are used for minimizing this noise.

As the noise is internally generated, external shielding cannot help in reducing this noise. Also since the noise has a wide frequency band, selective filtering is ineffective in reducing the magnitude of this noise.

A second type of noise is generated internally by short time electrical events within an active device like a transistor. In semi-conductor devices charges cross junctions (for example  $p-n$  junction) within the device, and therefore move from one energy level to another. This movement results in acceleration of charges which generate electromagnetic disturbances, which in turn produce an internally generated noise. Since, the period of transition from one energy level to another is short, the period of acceleration is also short, and hence the resulting frequency spectrum of generated noise is wideband. The same type of noise is generated as electrons pass through various electrode areas of a vacuum tube which have different electric field intensities thereby causing electrons to accelerate and cause electromagnetic disturbances.

The internally generated noise in semi-conductor and vacuum tube devices on account of random movement of charges is called **Shot Noise**. Not much can be done to reduce the internally generated noise in semi-conductor and vacuum tube devices as this noise is on account of acceleration of charges, a phenomenon so essential for operation of these devices. Selective filtering may be helpful to some extent in reduction of this noise.

Changing electric fields in the region between plates of a capacitor and changing magnetic fields around in an inductor are other sources of internally generated noise. However, in these cases, the electric and magnetic

Fig. 2.3. An amplifier for illustrating the different types of noise signals. The output signal is amplifier gain times the input signal plus a noise signal. One of the possible sources of noise is on account of internal components of the amplifier like resistors, capacitors and transistors etc. Therefore the noise in this case is generated inside the amplifier and therefore is called **Generated noise**. The internally generated noise is on account of components like resistors, capacitors, transistors etc. as stated above.



fields change in an orderly manner and therefore the noise signal has a fixed frequency and hence its magnitude can be reduced by using a filter tuned to this frequency.

(ii) **Conducted Noise.** The power supply to the amplifier could be the source of noise since it may have spikes, ripples or random deviations that are conducted to the amplifier circuit through power wiring. This type of noise is called **Conducted Noise**.

One of the most common sources for conducted noise is the 50 Hz power supply and the harmonics contained in it. The conducted noise can be reduced to use filters in the leads to trap out the noise.

(iii) **Radiated Noise.** There may be electric or magnetic fields or disturbances in the environments around the amplifier because of which unwanted signals are radiated into the interior of the amplifier and this is called **Radiated Noise**.

One of the common sources of radiated noise are the electromagnetic impulses radiated from the ignition wiring of spark plugs. The radiated noise can be reduced by proper shielding.

**2.13.3. Johnson Noise.** As stated earlier, Johnson noise is the thermally generated noise in a resistor.

The noise power  $P_n$  generated in a conductor is

$$P_n = kT \Delta f \quad \dots(2.11)$$

where  $k =$  Boltzmann constant  
 $= 1.38 \times 10^{-23} \text{ J/K},$

$T =$  absolute temperature of resistor ; K,

$\Delta f =$  bandwidth of noise signal ; Hz.

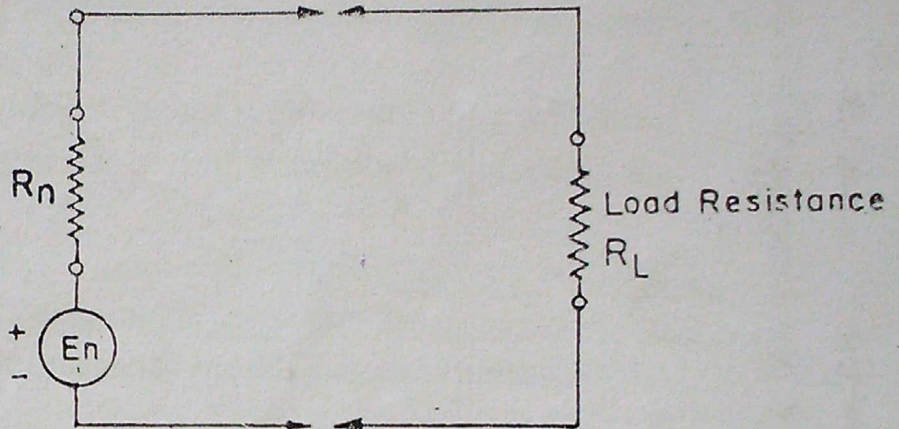


Fig. 2.4. Circuit representation of noise signals.

The noise generation system can be represented by a voltage source of magnitude in series with an equivalent resistance,  $R_n$ , as shown in Fig 2.4.

If the noise generator is connected to an external load resistance,  $R_L$ , the noise energy will be transferred to the load. Under conditions of maximum power transfer ( $R_L = R_n$ ), the noise power delivered to load is,

$$P_{nL} = \frac{E_n^2}{4 R_n} \quad \dots(2.12)$$

or

$$E_n^2 = 4 R_n P_{nL} \\ = 4 kT R_n \Delta f$$

or noise voltage

$$E_n = 2\sqrt{kTR_n \Delta f} \quad \dots(2.13)$$

Suppose two resistors  $R_1$  and  $R_2$  are connected in series, the total noise is the sum of the noises of individual resistors.

$$E_n^2 = E_{n1}^2 + E_{n2}^2 \\ = 4kTR_1 \Delta f + 4kTR_2 \Delta f \\ = 4kTR_s \Delta f$$

or

$$E_n = 2\sqrt{kTR_s \Delta f} \quad \dots(2.14)$$

where

$$R_s = R_1 + R_2$$

Similarly, the total noise voltage for parallel connected resistors is,

$$E_n = 2\sqrt{kTR_p \Delta f} \quad \dots(2.15)$$



where

$V_n$  = resultant voltage of parallel connected resistors.

Now,

$$\frac{S}{N} = \frac{\text{signal power}}{\text{noise power}} = \frac{V_s^2/R}{V_n^2/R} = \frac{V_s^2}{V_n^2} \quad \dots(2.16)$$

where  $V_s$  and  $V_n$  are respectively the signal and noise voltages.

**2.13.4. Power Spectral Density.** The power spectral density  $S_n$  is defined as the noise power per unit of frequency bandwidth. Therefore from Eqn. 2.11,

$$S_n = \frac{P_n}{\Delta f} = kT \quad \dots(2.17)$$

Power Spectrum Density represents noise energy generated per cycle of vibration present in the generating conductor.

**2.13.5 Noise Factor and Noise Figure.** The noise factor is defined as:

$$F = \frac{S/N \text{ at input}}{S/N \text{ at output}} \quad \dots(2.18)$$

Noise factor measurements are important because they are a measure of noise added to a signal by a device in the measurement system. If noise factor is expressed in decibels, it is called noise figure.

$$\text{Noise figure} \quad nf = 10 \log F \quad \dots(2.19)$$

The measurement of noise figure is most meaningful measurement for amplifiers, transistors and vacuum tubes since it is a measure of the noise generated within the device. The measurement of noise figure requires the use of known source of noise.

The noise figure from Eqn. 2.19 is,

$$\begin{aligned} nf &= 10 \log \left[ \frac{\text{noise voltage at output with noise at input}}{\text{noise voltage at output with no noise at input}} \right] \\ &= 10 \log \frac{V_n}{V_0} \quad \dots(2.20) \end{aligned}$$

where

$V_n$  = output noise voltage with a noise source injected into the input,

$V_0$  = output noise voltage without noise at input.

The difference between the two voltages represents the noise added by the device under test.

**Example 2.7.** An amplifier whose bandwidth is 100 kHz has a noise power spectral density input of  $7 \times 10^{-21}$  J. If the input resistance is 50 k $\Omega$  and the amplifier gain 100, what is the noise output voltage?

**Solution.** From Eqn. 2.17, power density spectrum

$$S_n = kT = 7 \times 10^{-21}$$

Input resistance

$$R = 50 \times 10^3 \Omega, \quad \text{Bandwidth} = 100 \times 10^3 \text{ Hz}$$

From Eqn. 2.13 noise voltage at the input

$$\begin{aligned} E_n &= 2 \sqrt{kTR \Delta f} \\ &= 2 \sqrt{7 \times 10^{-21} \times 50 \times 10^3 \times 100 \times 10^3} = 11.83 \mu\text{V} \end{aligned}$$

$\therefore$  Noise voltage at the output

$$\begin{aligned} &= E_n \times \text{gain of amplifier} \\ &= 11.83 \times 100 \mu\text{V} = 1.183 \text{ mV.} \end{aligned}$$

**Example 2.8.** A low pass filter has an input  $S/N$  of 20. The input voltage is 3 mV. Calculate the noise voltage.

**Solution.** From Eqn. 2.16, the signal to noise ratio

$$\frac{S}{N} = \frac{V_s^2}{V_n^2}$$



## 1.7 Input-output configuration of measuring instruments

An instrument performs an operation on an input quantity (called measurand or process variable and designated  $i$ ) to provide an output (called measurement and designated  $o$ ). Accordingly the performance of the instrument can be stated in terms of an operational transfer function  $G$ . The input-output relationship is characterised by the operation  $G$  such that:  $o = G i$

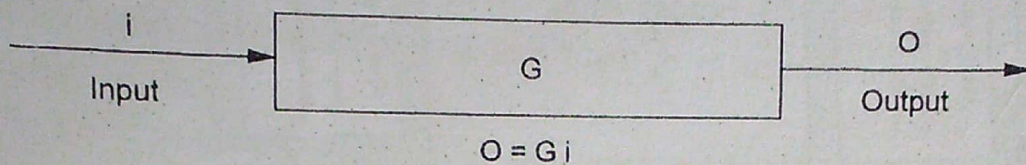


Fig 1.9 Input-output relation of a measurement system

The various inputs to a measurement system can be classified into three categories :

1. **Desired input** : a quantity that the instrument is specifically intended to measure. The desired input  $i_D$  produces an output component according to an input-output relation symbolised by  $G_D$ ; here  $G_D$  represents the mathematical operation necessary to obtain the output from the input.
2. **Interfering input** : a quantity to which the instrument is unintentionally sensitive. The interfering input  $i_I$  would produce an output component according to input-output relation symbolised by  $G_I$ .
3. **Modifying input** : a quantity that modifies the input-output relationship for both the desired and interfering inputs. The modifying input  $i_M$  would cause a change in  $G_D$  and/or  $G_I$ . The specific manner in which  $i_M$  affects  $G_D$  and  $G_I$  is represented by the symbols  $G_{MD}$  and  $G_{MI}$  respectively.

A block diagram of these various aspects has been illustrated in Fig 1.10.

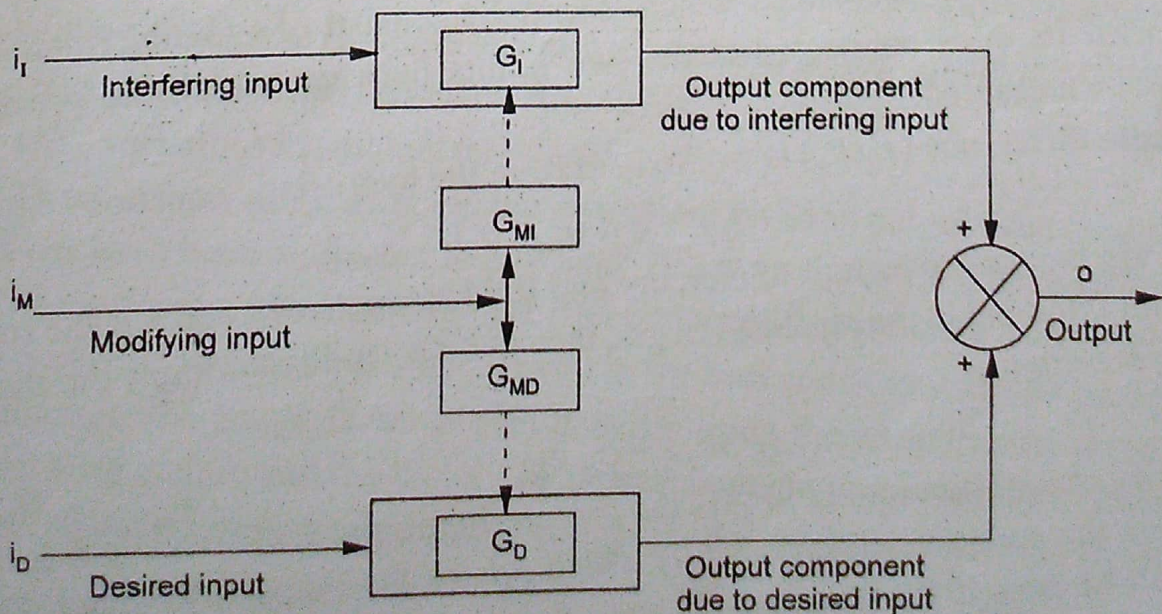


Fig 1.10 Generalised input-output configuration



The output of the measuring apparatus is the instantaneous algebraic sum of the output component due to the desired, interfering and modifying inputs.

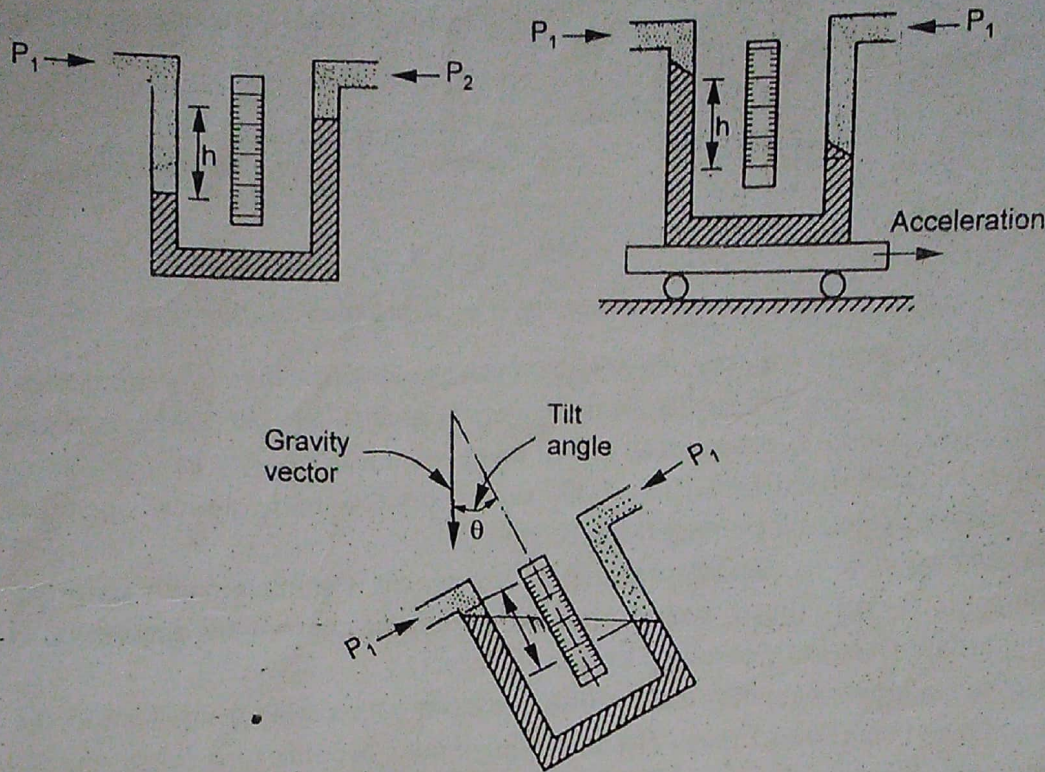


Fig 1.11 Spurious inputs for a manometer

Consider a differential manometer which essentially consists of a U-tube filled with mercury and with its ends connected to the two points between which the pressure differential is to be measured.

The pressure difference ( $P_1 - P_2$ ) is worked out from the hydrostatic equation:

$$(P_1 - P_2) = g h (\rho_m - \rho_f)$$

where  $\rho_m$  and  $\rho_f$  are the mass densities of mercury and fluid respectively, and  $h$  is the scale reading. If the fluid flowing in the pipeline is a gas, then  $\rho_f \ll \rho_m$  and accordingly the above identity can be re-written as

$$(P_1 - P_2) = \rho_m g h$$

Here the differential pressure ( $P_1 - P_2$ ) is the desired input; the scale reading  $h$  is the output and  $\rho_m g$  is the parameter (scale factor) which relates the output with input. The scale reading will be zero if the pressures  $P_1$  and  $P_2$  are equal, i.e., when the pressure differential is zero. This simple result is, however, not true for the following cases:-

1. The manometer is placed on a wheel which is subjected to acceleration. A pressure differential gets created due to acceleration and the scale indicates a reading even though the pressures  $P_1$  and  $P_2$  at the two ends are equal.

The acceleration then constitutes the *interfering input*

2. The manometer has an angular tilt, i.e., it is not properly aligned with the



direction of gravitational force. An output will result even when there is no pressure difference.

Here the angular tilt acts as the *interfering input*

The scale factor establishes the input - output relation, and this gets modified due to :

- (a) temperature variations which change the value of density of mercury
- (b) change in gravitational force due to change in location of manometer

A change in ambient temperature also changes the lengths of calibrated scale and that also leads to modification of proportionality factor.

The temperature variations and change in gravitational force then constitute the *modifying inputs*.

As another example, consider an electrical resistance strain gauge set-up used for the measurement of strain induced in an element due to mechanical loading. When the gauge is strained, the change in output voltage is

$$dV_0 = \frac{V_s}{4} \times \frac{dR}{R}$$

where  $V_s$  is the battery supply voltage and  $dR$  is the change in resistance of the strain gauge which constitutes one arm of the Wheat stone bridge circuit (Refer Art 11.9.2, Deflection mode). Here it has been presumed that all the limbs of the Wheat-stone bridge have equal resistances, *i.e.*,

$$R_1 = R_2 = R_3 = R_4 = R$$

In terms of applied strain  $\epsilon$  and the strain gauge factor  $F$ ,

$$dV_0 = \frac{V_s}{4} F \epsilon$$

Evidently the change in output voltage  $dV_0$  is directly proportional to the applied strain.

In the above cited example of strain measurement, the typical interfering inputs are :

- (i) stray electrostatic and electric fields
- (ii) variations in ambient temperature

These effects cause drift and the instrument gives an output even though the element might not have been subjected to any load, *i.e.*, even though the desired input (strain) is zero. The influence of temperature is better understood as it causes a change in length (and hence a change in gauge resistance) even in the absence of any mechanical strain.

Further, the gauge factor  $F$  also depends upon the temperature. Accordingly, the ambient temperature and the battery voltage change the proportionality constant between the desired input and the bridge output voltage, and as such constitute the typical modifying inputs.

Often it is desired to nullify or reduce the effect of spurious inputs on the indicated output of an instrument. The various methods used to accomplish this task are:

1. **Signal filtering** : Certain elements called filters are introduced into the instrument



in the path of spurious inputs. The filters are designed to be selective, *i.e.*, they pass wanted signal and reject or considerably decrease the unwanted signal. The filters can be introduced into the measurement system at one of its three stages (input, intermediate and output stage); the choice depends upon the application (Refer Article 5.8 Chapter 5).

A similar purpose is achieved by

- (i) correct grounding and electrical shielding of the instrument
- (ii) mounting a vibration sensitive instrument on a spring-mass damping system

**2. Compensation by opposing inputs :** This method consists in intentionally introducing into the instrument certain interfering and/or modifying inputs that tend to cancel the bad effects of unavoidable spurious inputs. For example, the effect of ambient temperature changes at the capillary and bourdon tube of filled-in-system thermometers can be reduced or compensated by employing case compensation, full compensation or self-compensating capillary tubes (Refer Article 10.6.4, Chapter 10).

The influence of ambient temperature in the act of strain measurement by an electrical resistance strain gauge is nullified by the use of a temperature compensating (dummy) strain gauge. The dummy gauge is identical to the active gauge (a gauge mounted on the test-piece and subjected to loading) and the two form a matched pair. However, the dummy gauge is bonded to a separate, unstrained component identical to that of the loaded member. Evidently the dummy gauge remains unstrained throughout the test-run and suffers changes in resistance due to temperature only. The active and the dummy gauges are connected in the adjacent arms of the Wheatstone bridge, and the bridge would be temperature compensated as long as both the gauges are at the same temperature. The output voltage would then be a function of only the direct load (applied strain) and the temperature effects would get cancelled (Refer Article 11.10; Chapter 11).

Strain  
Measure  
ment  
[Temp.  
Compensation]

**3. Output correction :** The method involves in calculating the corrections which may be added to or subtracted from the indicated output so as to leave only that component which is associated with the desired input.

The glass thermometers are generally graduated for total immersion of bulb and stem. Whilst in use, the thermometer may only be partially immersed. The error proportional to the exposed portion of the stem (stem-emergence effect) is evaluated and necessary correction is applied to the indicated value of temperature. Likewise in strain gauge arrangement, a knowledge of the temperature coefficient of resistance, temperature sensitivity of gauge and the operating temperature helps to obtain the correction to the output.

The effect of spurious inputs are also reduced by using :

- (1) high amplification and a stable/accurate feed back device.
- (2) instrument elements which are inherently sensitive only to the desired input. For example, the strain gauge may be made of a material with a very low temperature coefficient of resistance and temperature independent gauge factor.