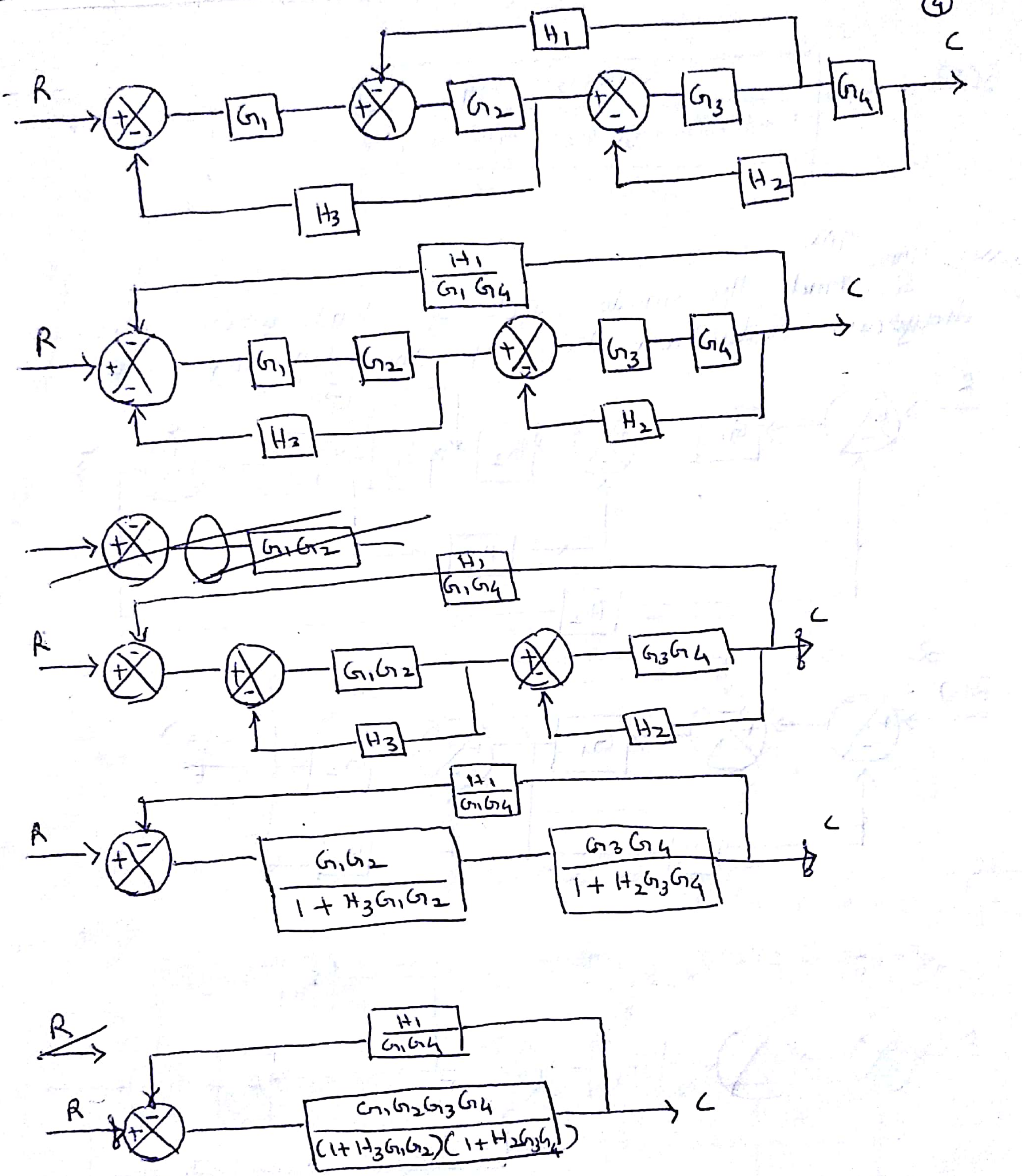
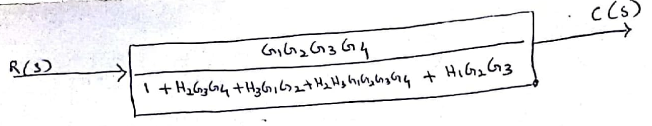
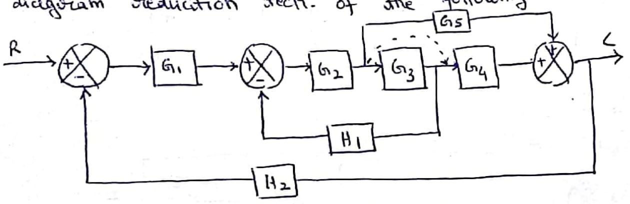


$$\frac{\dfrac{G_1G_2G_3G_4}{1+H_2G_3G_4+H_3G_1G_2+H_2H_3G_1G_2G_3G_4}}{1+\dfrac{H_1}{G_1G_4}\times\dfrac{G_1G_2G_3G_4}{1+H_2G_3G_4+H_3G_1G_2+H_2H_3G_1G_2G_3G_4}}$$

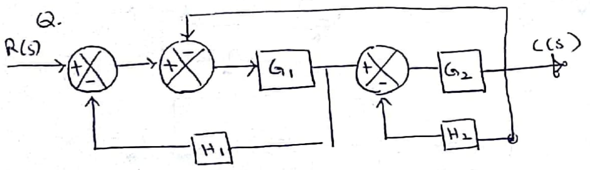


H.W. Univ. 7M

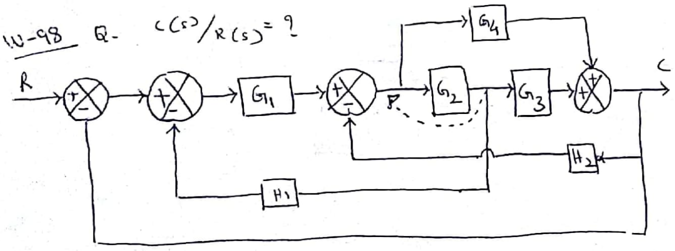
Q. Find the single block equivalent using block diagram reduction tech. of the following one:



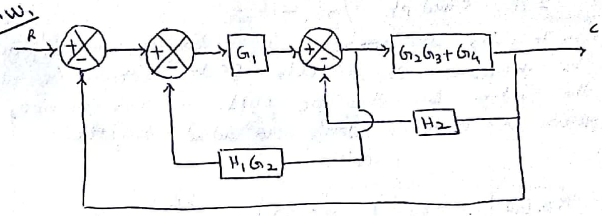
Q.

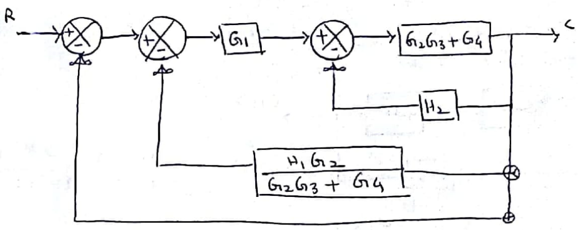


W-98 Q. $C(s)/R(s) = ?$



H.W.



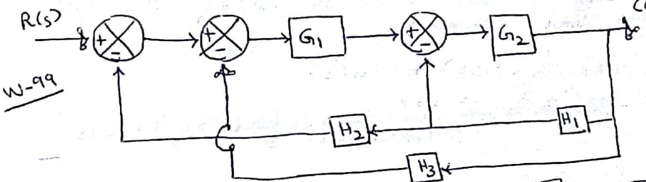


Check it

$$\boxed{\frac{C(s)}{R(s)} = \frac{G_1G_2G_3 + G_1G_4}{1+G_1G_2G_3+G_1G_4+G_1G_2H_1+G_2G_3H_2+G_4H_2}}$$

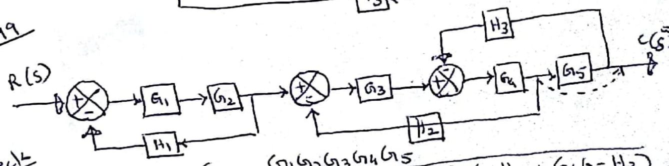
Ex: Find $\frac{C(s)}{R(s)} = ?$ Ans: T.F. $= \dfrac{G_1G_2}{1+G_2H_1+G_1G_2H_3+G_1G_2H_1H_2}$

W-99



S-99

Ex:

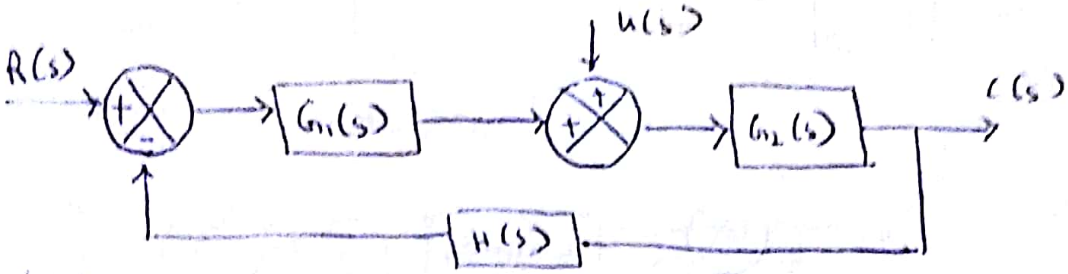


Check it

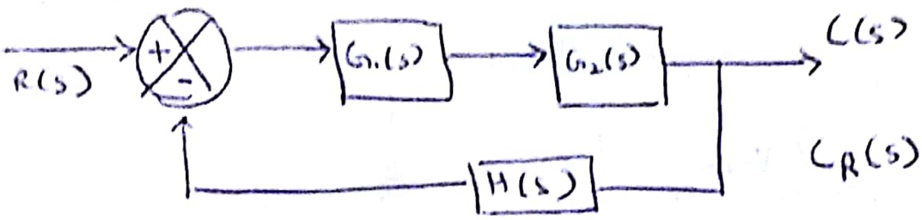
$$\frac{C}{R} = \frac{G_1G_2G_3G_4G_5}{(1+G_1G_2H_1)(1+G_3G_4H_2+G_4G_5H_3)}$$

Multiple I/p / Multiple o/p :-

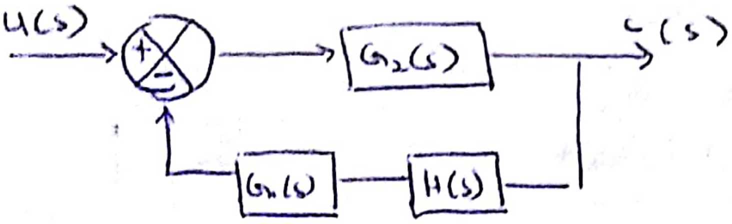
When multiple i/ps are present in a linear system, each i/p can be treated independently of the others. Complete o/p of the system can then be obtd. by superposition, i.e., o/ps corres. to each i/p alone are added together.



$U(s) = 0$



$$C_R(s) = \frac{G_1(s)G_2(s)}{1 + G_1(s)G_2(s)H(s)} R(s)$$

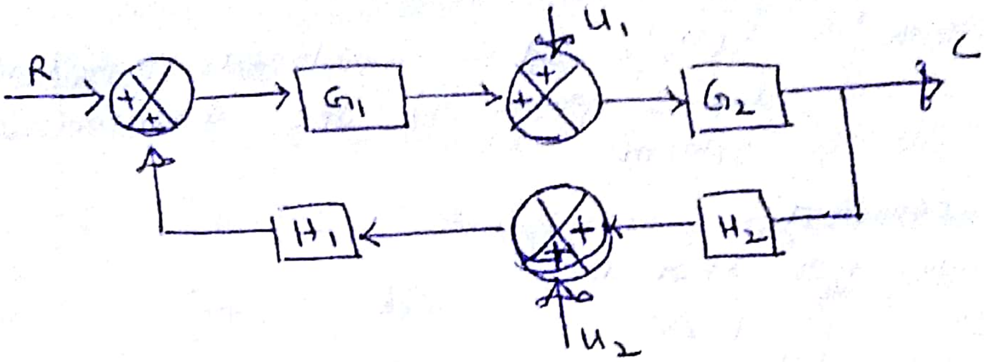


$$C_U(s) = \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)} U(s)$$

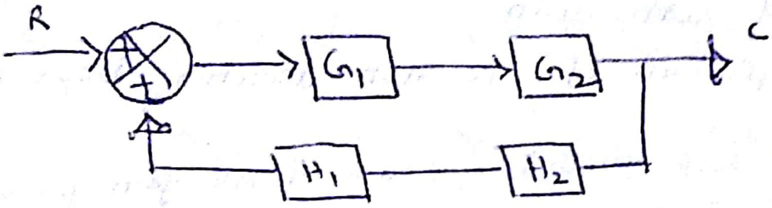
The response to the simultaneous application of R(s) & U(s) can be obtd. by adding the two individual response-

$$C(s) = C_R(s) + C_U(s)$$

$$= \frac{G_2(s)}{1 + G_1(s)G_2(s)H(s)} \left[G_1(s)R(s) + U(s)\right]$$

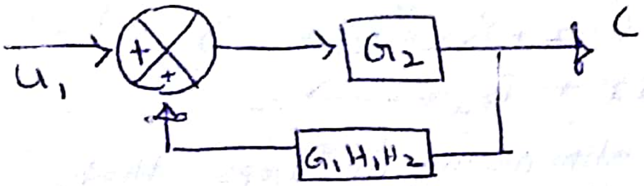


$u_1 = u_2 = 0$



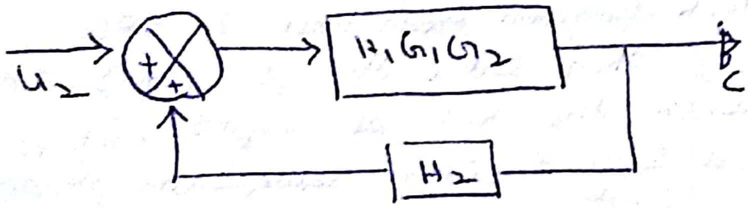
$$C_R(s) = \frac{G_1 G_2}{1 - G_1 G_2 H_1 H_2} R(s)$$

$u_2 = R = 0$



$$C_{u_1}(s) = \frac{G_2}{1 - G_1 G_2 H_1 H_2} u_1(s)$$

$u_1 = R = 0$



$$C_{u_2}(s) = \frac{G_1 G_2 H_1}{1 - G_1 G_2 H_1 H_2} u_2(s)$$

$$C = C_R(s) + C_{u_1}(s) + C_{u_2}(s)$$

$$= \frac{G_1 G_2 R(s) + G_2 u_1(s) + G_1 G_2 H_1 u_2(s)}{1 - G_1 G_2 H_1 H_2}$$

$$\boxed{C = \frac{G_2}{1 - G_1 G_2 H_1 H_2}\left[G_1 R(s) + u_1(s) + G_1 H_1 u_2(s)\right]}$$

Scanned by CamScanner

Signal Flow Graph :- **Defn** :- It is a pictorial representation of a system & it displays graphically the transmission of signals in a system.

Mason's Gain Equation :-

The G. Formula for SFG is

$$\frac{C}{R} = T = \frac{\sum_{i=1}^{n} P_i \Delta_i}{\Delta}$$

P_i = ith forward path gain

P_{jk} = jth possible product of k non-touching loops gain

$\Delta \rightarrow$ 1 − (sum of loop gains) + (sum of all gain products of 2 non-touching loops) − (sum of all gain products of 3 non-touching loops) + ---

$$\therefore \Delta = 1 - (P_{11} + P_{21} + \cdots) + (P_{12} + P_{22} + P_{32} + \cdots) - (P_{13} + P_{23} + P_{33} + \cdots) \cdots$$

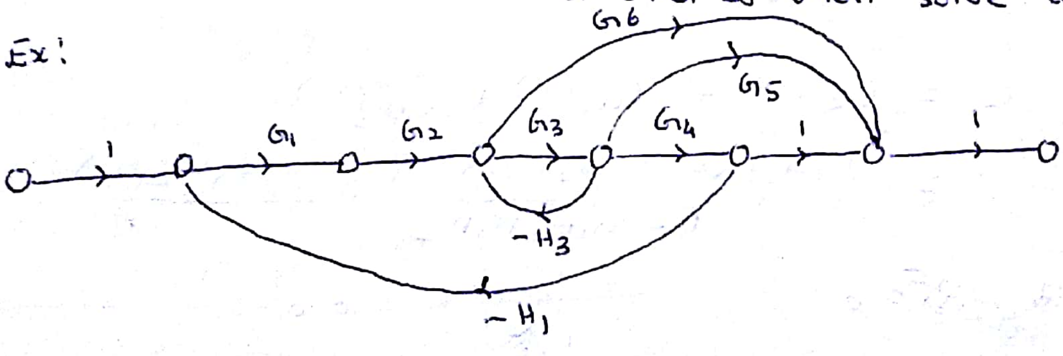
Δ_i = Δ evaluated after eliminating all loops that touch the kth forward path.

Problems : Type 1 (on the basis of SFG only)

Type 2 (Block diagram reduction to SFG & then solve the problem)

Type 3 (with the help of eqns obtain a SFG & then solve the problem)

Ex:



(1) There are 3 forward paths

$P_1 = G_1 G_2 G_3 G_4$

$P_2 = G_1 G_2 G_6$

$P_3 = G_1 G_2 G_3 G_5$

(2) There are 2 loops

$P_{11} = -G_3H_3 \quad P_{21} = -G_1G_2G_3G_4H_1$

(3) Both loops touch. Hence higher order terms disappear.

P_{12}, P_{22}, P_{32} - - - - are absent

$P_{13}, P_{23}, P_{33}, P_{43}$ - - - are absent.

(4) $\Delta = 1 - (P_{11} + P_{21}) = 1 + G_3H_3 + G_1G_2G_3G_4H_1$

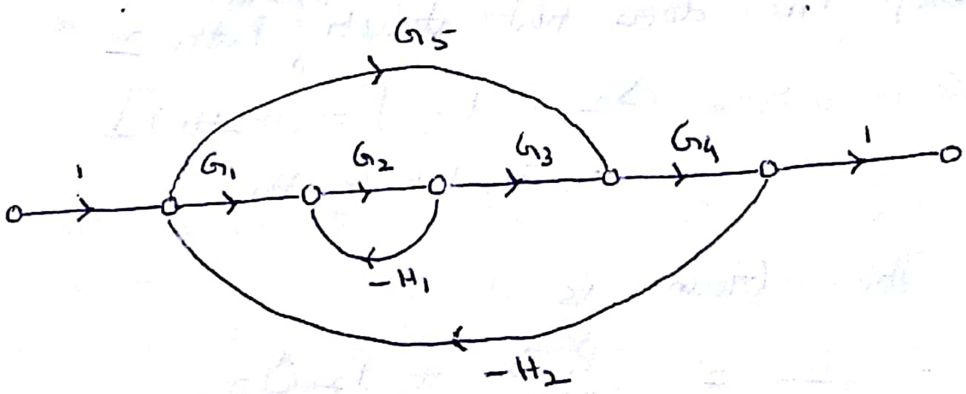
(5) Take P_1. All loops touch it. $\Delta_1 = 1 - (0) = 1$

Also, P_2 & P_3. All loops touch it. $\Delta_2 = \Delta_3 = 1$

(6) $T = \frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2 + P_3\Delta_3}{\Delta}$

$$\boxed{T = \frac{G_1G_2G_3G_4 + G_1G_2G_6 + G_1G_2G_3G_5}{1 + G_3H_3 + G_1G_2G_3G_4H_1}}$$

Ex:



(1) $P_1 = G_1G_2G_3G_4$

$P_2 = G_4G_5$

(2) $P_{11} = -G_2H_1 \quad P_{21} = -G_4G_5H_2$

$P_{31} = -G_1G_2G_3G_4H_2$

Scanned by CamScanner

(3) Out of the above 3 loops P_{11} & P_{21} are non-touching

$$P_{12} = P_{11} \times P_{21} = (-G_2H_1) \times (-G_4G_5H_2)$$
$$= G_2G_4G_5H_1H_2$$

(4) There are no 3-non-touching loops. P_{13}, P_{23} ... terms are absent.

(5)
$$\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (P_{12})$$
$$= 1 - [(-G_2H_1) + (-G_4G_5H_2) + (-G_1G_2G_3G_4H_2)]$$
$$+ G_2G_4G_5H_1H_2$$
$$= 1 + G_2H_1 + G_4G_5H_2 + G_1G_2G_3G_4H_2 + G_2G_4G_5H_1H_2$$

(6) All loops touch P_1. $\therefore \Delta_1 = 1 - (0) = 1$

Loop "P_{11}" does not touch forward Path 2

$$\therefore \Delta_2 = 1 - [-(G_2H_1)]$$
$$= 1 + G_2H_1$$

(7) The Gain is

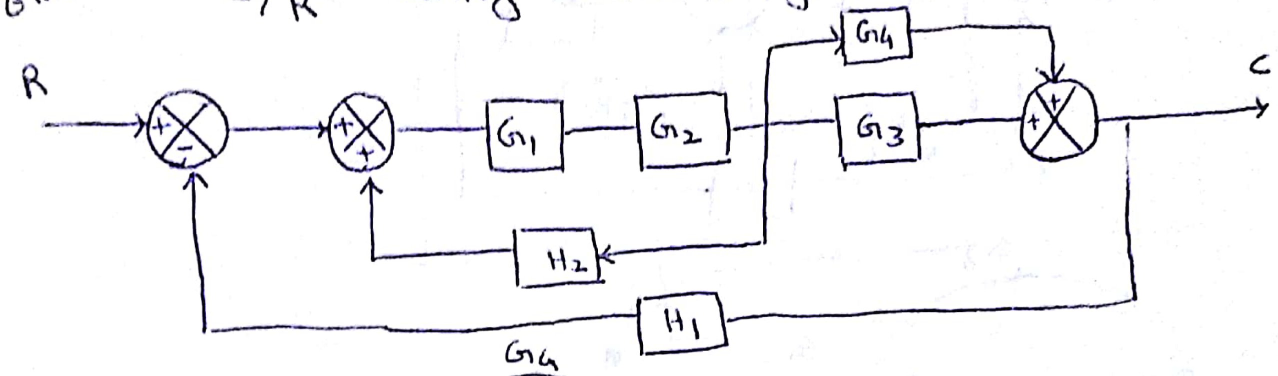
$$T = \frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

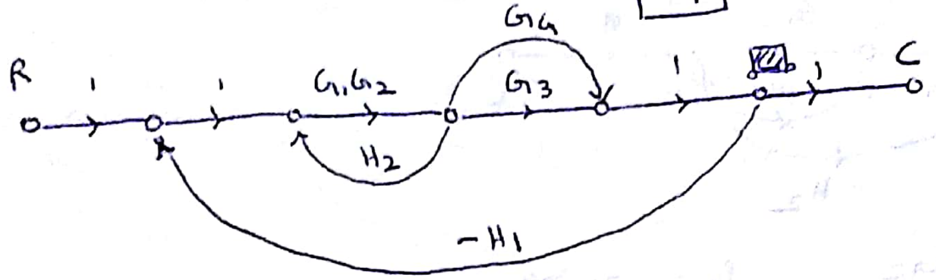
$$\boxed{T = \frac{G_1G_2G_3G_4 + G_4G_5(1 + G_2H_1)}{1 + G_2H_1 + G_4G_5H_2 + G_1G_2G_3G_4H_2 + G_2G_4G_5H_1H_2}}$$

(Ans.)

Ex:
S-2000
GMKS

Convert the block diagram shown in fig into a signal flow graph & determine the gain C/R using Mason's gain Formula-





(1) $P_{\Pi} = P_1 = G_1G_2G_3 \qquad P_2 = G_1G_2G_4$

(2) $P_{11} = G_1G_2H_2 \quad P_{21} = -H_1G_1G_2G_3 \quad P_{31} = -H_1G_1G_2G_4$

P_{21}, P_{22}, P_{23} --- terms are absent

(3) $\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (0)$

$= 1 - G_1G_2H_2 + G_1G_2G_3H_1 + G_1G_2G_4H_1$

(4) All loops touch P_1 $\therefore$ $\Delta_1 = 1 - (0) = 1$

— " —— " — P_2 $\therefore$ $\Delta_2 = 1 - (0) = 1$

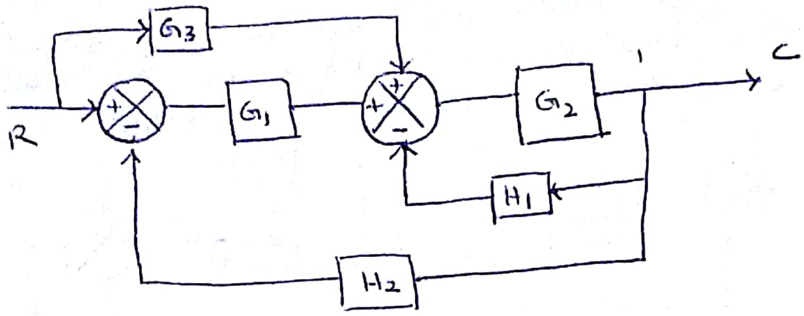
also

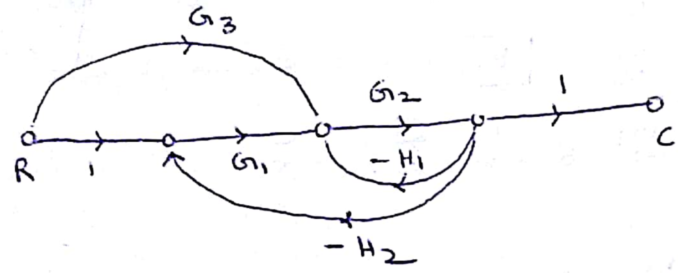
(5) The Gain is

$$T = \frac{C}{R} = \frac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$$

$$= \frac{G_1G_2G_3 + G_1G_2G_4}{1 - G_1G_2H_2 + G_1G_2G_3H_1 + G_1G_2G_4H_1}$$

Ex:





① $P_1 = G_1 G_2$ $\qquad P_2 = G_2 G_3$

② $P_{11} = -G_2 H_1$ $\qquad P_{21} = -G_1 G_2 H_2$

③ There are no 2 non-touching loops

④ $\Delta = 1 - (P_{11} + P_{21}) + (0)$

$= 1 + G_2 H_1 + G_1 G_2 H_2$

⑤ All loops touch the forward paths "P_1" & "P_2"

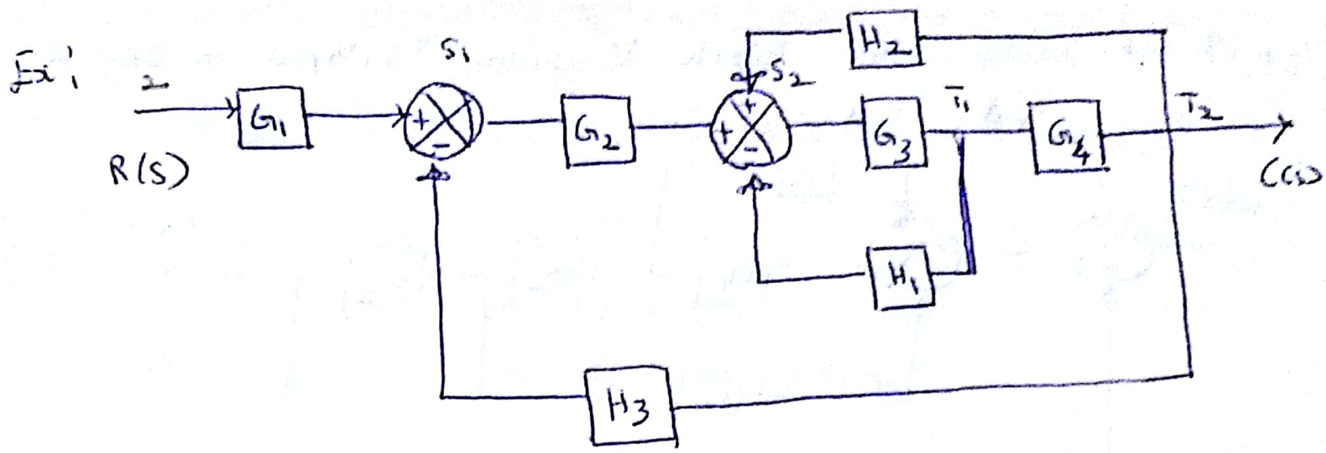
$\therefore \Delta_1 = 1 - (0) = 1$

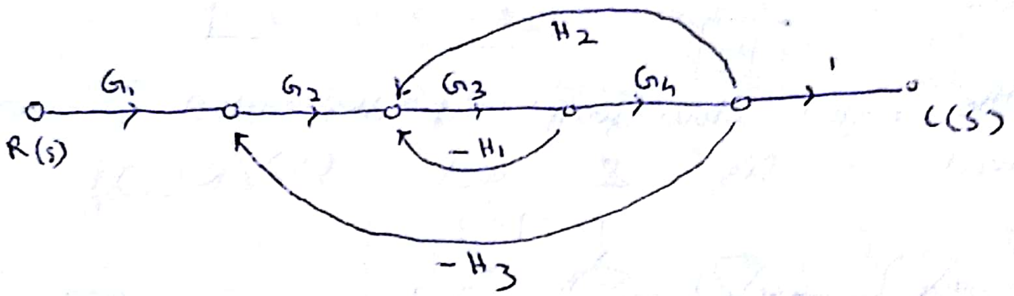
$\Delta_2 = 1 - (0) = 1$

⑥ $$T = \frac{C}{R} = \frac{P_1 \Delta_1 + P_2 \Delta_2}{\Delta}$$

$$= \frac{G_1 G_2 + G_2 G_3}{1 + G_2 H_1 + G_1 G_2 H_2}$$

Ex:





(1) $P_1 = G_1G_2G_3G_4$ (2) $P_{11} = -G_3H_1$ $P_{21} = -G_2G_3G_4H_3$

$P_{31} = G_3G_4H_2$

(3) All loops touch each other & also the forward path

i.e. P_{12}, P_{22}, P_{32} --- are zeros.

P_{13}, P_{23}, P_{33} --- are zeros.

(4) $\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (0)$

$= 1 + G_3H_1 + G_2G_3G_4H_3 - G_3G_4H_2$

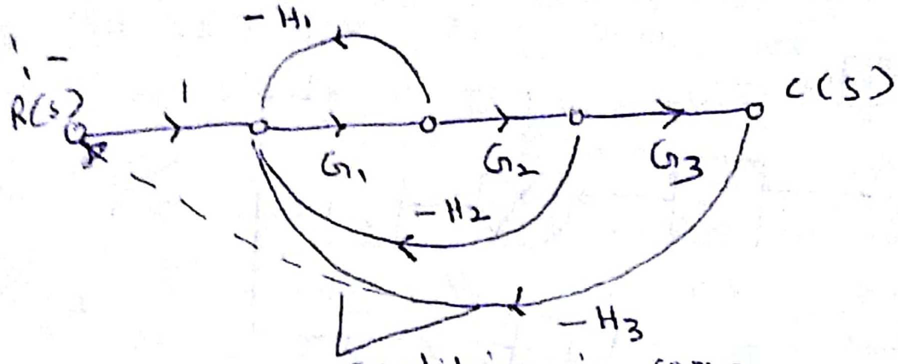
(5) All loops touch the forward path

$\therefore \Delta_1 = 1 - (0) = 1$

$\therefore \frac{C}{R} = T = \frac{P_1\Delta_1}{\Delta}$

$$\boxed{T = \frac{G_1G_2G_3G_4}{1 + G_3H_1 + G_2G_3G_4H_3 - G_3G_4H_2}}$$

SFG :-

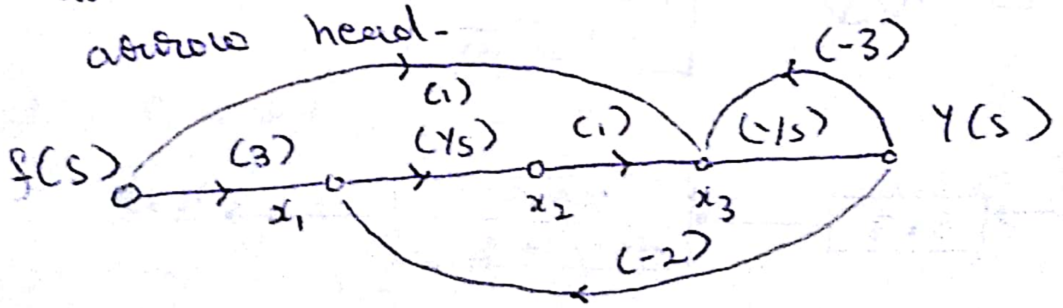


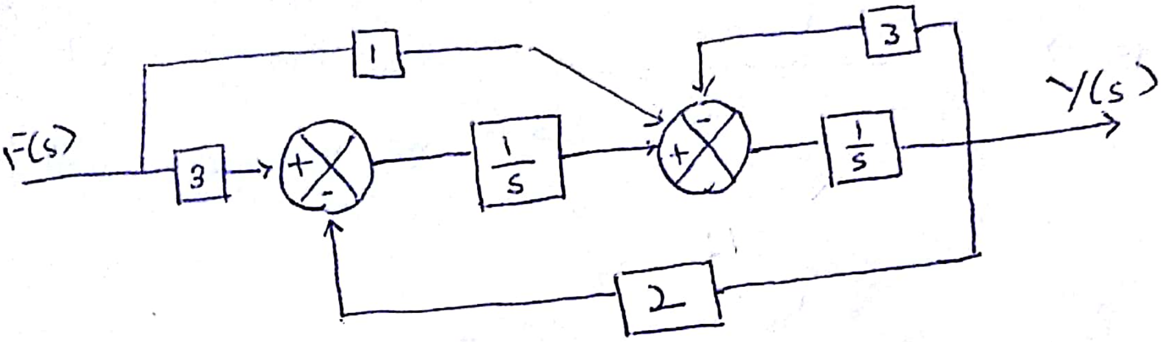
Condition is same

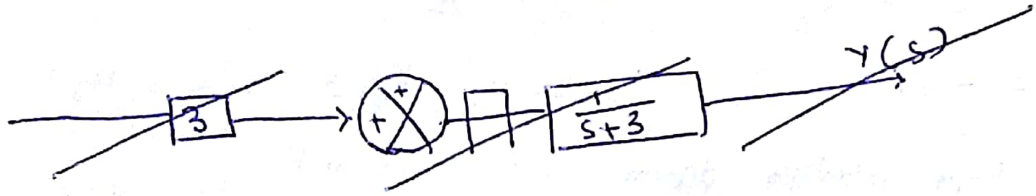
1. $P_1 = G_1 G_2 G_3$
2. $P_{11} = -G_1 H_1 \quad P_{21} = -G_1 G_2 H_2 \quad P_{31} = -G_1 G_2 G_3 H_3$
3. Higher order terms are absent.
4. $\Delta_1 = 1 - (0) = 1$
5. $\Delta = 1 - (P_{11} + P_{21} + P_{31}) + (0)$
   $= 1 + G_1 H_1 + G_1 G_2 H_2 + G_1 G_2 G_3 H_3$

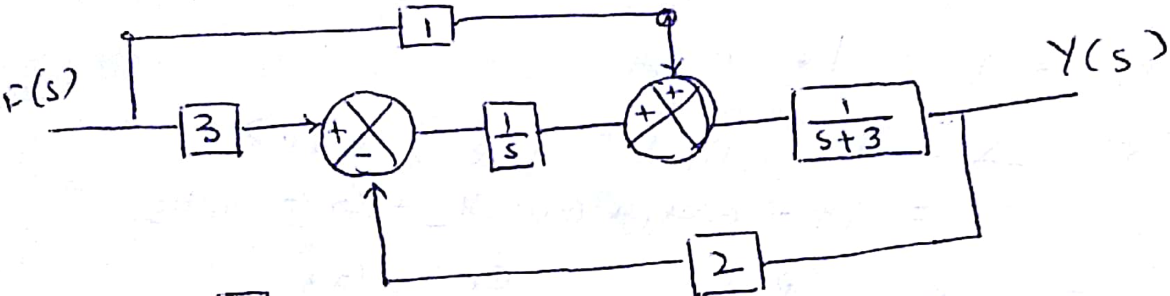
$$T = \frac{C}{R} = \frac{P_1 \Delta_1}{\Delta} = \frac{G_1 G_2 G_3}{1 + G_1 H_1 + G_1 G_2 H_2 + G_1 G_2 G_3 H_3}$$

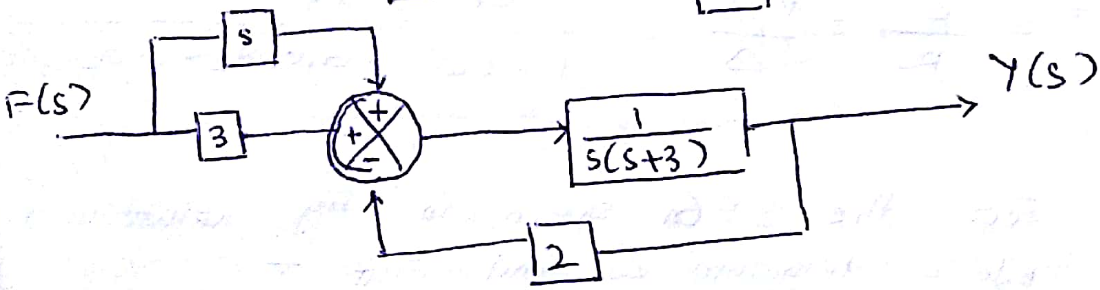
W-97 Ex: For the SFG shown in fig. construct the block diagram & find out T.F. $\frac{Y(s)}{F(s)}$ by —"—"— orah Branch gain are shown in round brackets near branch arrow head.

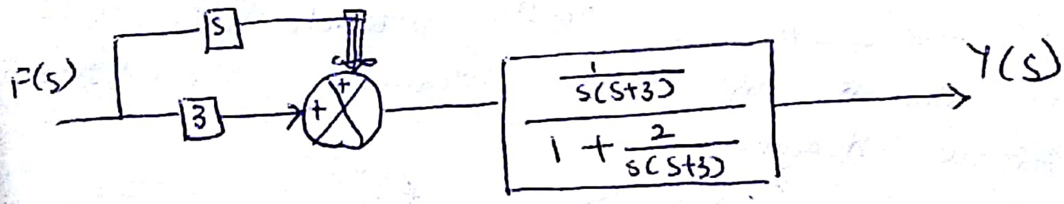


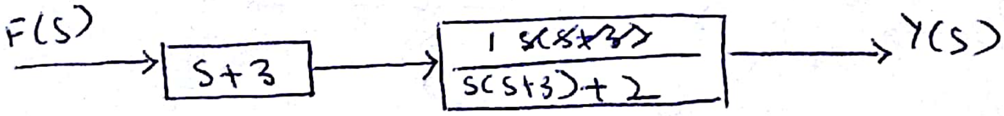












$$\boxed{\frac{Y(s)}{F(s)} = \frac{s+3}{s^2+3s+2}}$$

$\cancel{s(s+3)(s+3)}$

Terms Associated with SFG: W-2000

(i) I/p Node (Source): It is a node which has only outgoing branches.

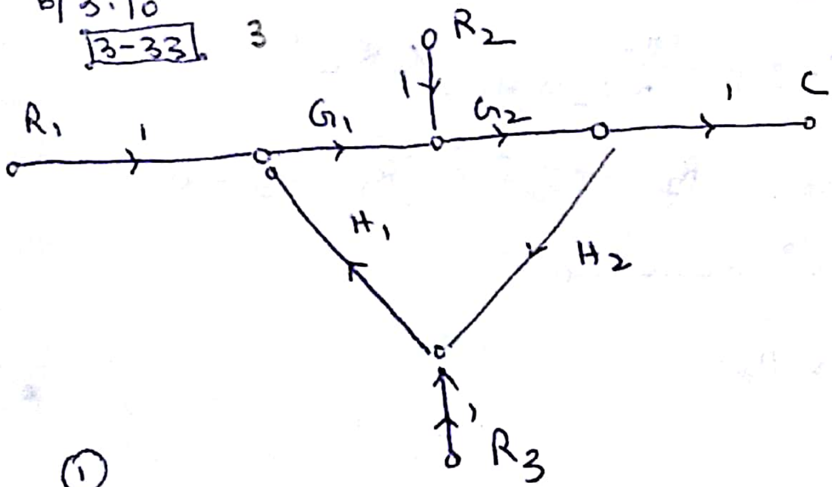
(ii) o/p Node: It is a node which has only incoming branches.

(iii) Path; ~~Alt~~ Forward Path; Loop; Path gain; Loop Gain

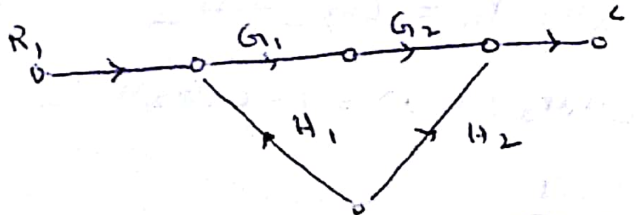
(iii) Path:- It is the traversal of connected branches in the dirn of the branch arrows such that no node is traversed more than once.

(iv) Loop:- It is the path which originates & terminates at the same node.

B/3.10
$\boxed{3-33}$ 3



① Consider R_1 alone: R_2 & $R_3 = 0$.



ⓐ $P_1 = G_1 G_2$ ⓑ $P_{11} = G_1 G_2 H_1 H_2$

ⓒ $\Delta = 1 - (G_1 G_2 H_1 H_2) + (0) = 1 - G_1 G_2 H_1 H_2$

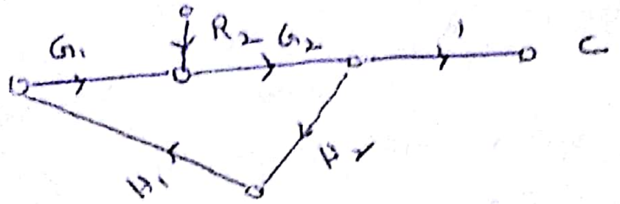
ⓓ $\Delta_1 = 1 - (0) = 1$

ⓔ $\dfrac{C}{R_1} = \dfrac{P_1 \Delta_1}{\Delta} = \dfrac{G_1 G_2}{1 - G_1 G_2 H_1 H_2}$

$C = \dfrac{G_1 G_2}{1 - G_1 G_2 H_1 H_2} \times R_1$

Scanned by CamScanner

(2) Consider "R_2" alone: $R_1 = R_3 = 0$



(a) $P_1 = G_2$ (b) $P_{11} = G_1 G_2 H_1 H_2$

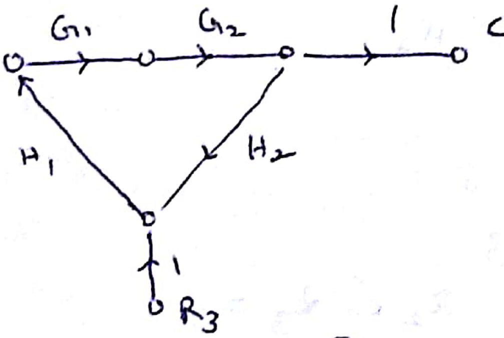
(c) $\Delta = 1 - (G_1 G_2 H_1 H_2) + (0) = 1 - G_1 G_2 H_1 H_2$

(d) $\Delta_1 = 1 - (0) = 1$

(e) $\frac{C}{R_2} = \frac{P_1 \Delta_1}{\Delta} = \frac{G_2}{1 - G_1 G_2 H_1 H_2}$

$$C = \frac{G_2}{1 - G_1 G_2 H_1 H_2} \times R_2$$

(3) Consider "R_3" alone: ~~$R_2 = R_{21} = 0$~~ $R_1 = R_2 = 0$



(a) $P_1 = H_1 G_1 G_2$ (b) $P_{11} = G_1 G_2 H_1 H_2$

(c) $\Delta = 1 - (G_1 G_2 H_1 H_2) + (0) = 1 - G_1 G_2 H_1 H_2$

(d) $\Delta_1 = 1 - (0) = 1$

(e) $\frac{C}{R_3} = \frac{P_1 \Delta_1}{\Delta} = \frac{H_1 G_1 G_2}{1 - G_1 G_2 H_1 H_2}$

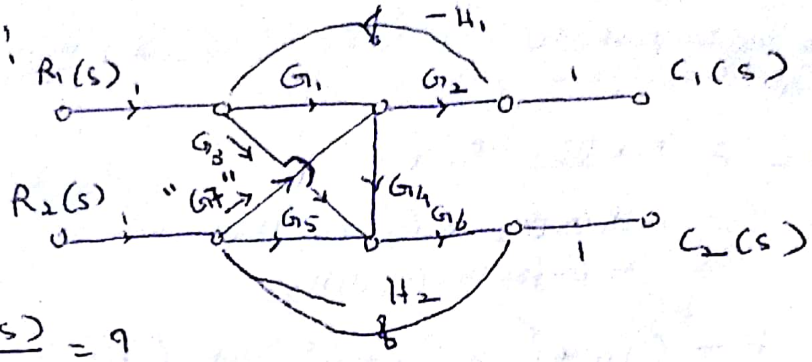
$$C = \frac{H_1 G_1 G_2}{1 - G_1 G_2 H_1 H_2} \times R_3$$

C = sum of o/ps due to each individual i/p,

$$\therefore \boxed{C = \frac{G_1 G_2 \times R_1 + G_2 \times R_2 + H_1 G_1 G_2 \times R_3}{1 - G_1 G_2 H_1 H_2}}$$

S-96

Ex:

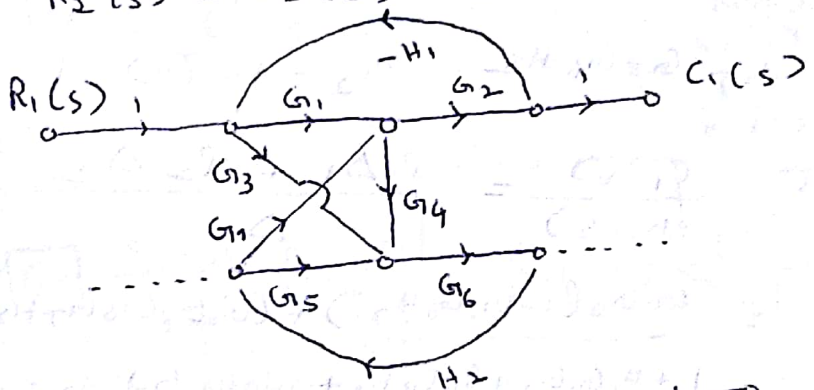


$$\frac{C_1(s)}{R_1(s)} = ?$$

$$\frac{C_2(s)}{R_2(s)} = ?$$

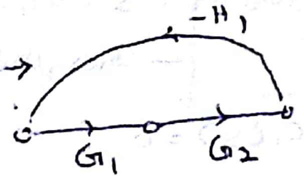
It is multi i/p & multi o/p system, so use super position principle

(1) $R_2(s) = C_2(s) = 0$

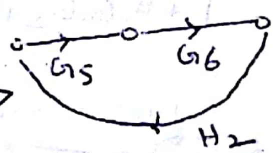


(3)

I →



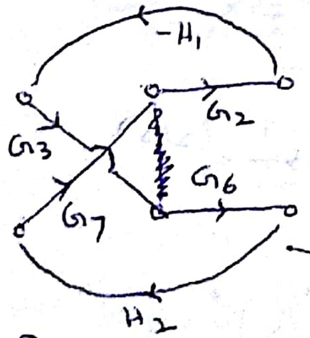
II →



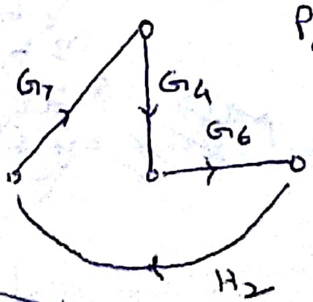
$P_{11} = -H_1G_1G_2$

$P_{21} = G_5G_6H_2$

III →



IV →



$P_{41} = G_4G_6G_7H_2$

$P_{31} = G_2(-H_1)G_3G_6H_2G_7 = -G_2G_3G_6G_7H_2H_1$

(2) "P_{10}" $= G_1G_2$ "P_{20}" $= G_3G_6H_2G_7G_2$
$= G_2G_3G_6G_7H_2$

(2)

(4) Find out all possible combination of 2 - non-touching loops.

$\cancel{P_{21}}$ $P_{12} = I \times II = P_{11} \times P_{21}$

$= -H_1G_1G_2 \times G_5G_6H_2$

$= -G_1G_2G_5G_6H_1H_2$

(5) $\Delta = 1 - (P_{11} + P_{21} + P_{31} + P_{41}) + (P_{12})$

$= 1 - \left[-(H_1G_1G_2) + G_5G_6H_2 - G_2G_3G_6G_7H_1H_2 + G_4G_6G_7H_2\right]$

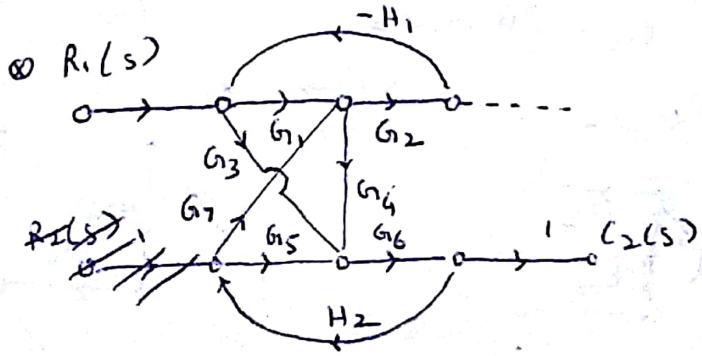
$+ \left[(-G_1G_2G_5G_6H_1H_2)\right]$

(6) $\Delta_1 = 1 - G_5G_6H_2 \qquad \Delta_2 = 1 - (0) = 1$

(7) $T = \dfrac{C_1(s)}{R_1(s)} = \dfrac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$

$$T = \frac{G_1G_2(1 - G_5G_6H_2) + (G_2G_3G_6G_7H_2) \times 1}{1 + H_1G_1G_2 + G_5G_6H_2 + G_2G_3G_6G_7H_1H_2 + G_4G_6G_7H_2 - G_1G_2G_5G_6H_1H_2}$$

(2) ~~$C_1(s) = R_2(s) = 0$~~ $R_2(s) = 0 \quad C_1(s) = 0$



"Δ" is same (check it)

(1) $P_1 = G_3G_6 \qquad \Delta_1 = 1 - (0) = 1$

$P_2 = G_1G_4G_6 \qquad \Delta_2 = 1 - (0) = 1$

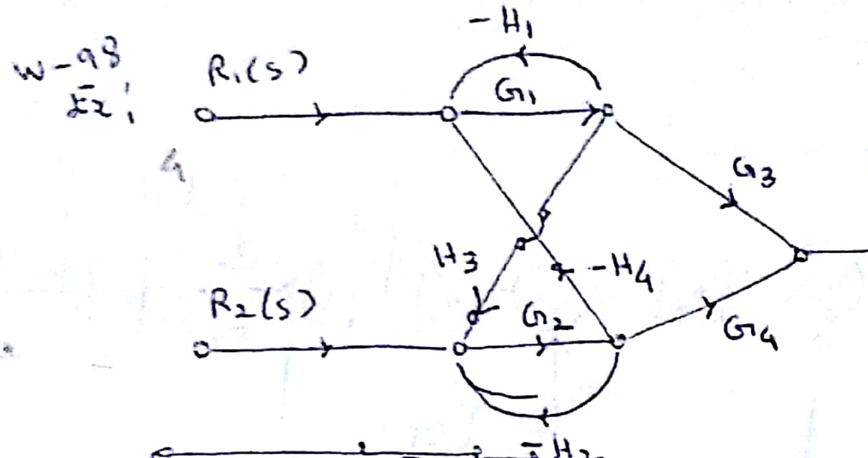
$T = \dfrac{C_2(s)}{R_1(s)} = \dfrac{P_1\Delta_1 + P_2\Delta_2}{\Delta}$

... "t" defined for

$$\frac{C_2(s)}{R_1(s)} = \frac{G_3G_6 + G_1G_4G_6}{1 + H_1G_1G_2 + G_5G_6H_2 + G_2G_3G_6G_7H_1H_2 + G_4G_6G_7H_2 - G_1G_2G_5G_6H_1H_2}$$

(Ans.)

W-98
Ex:



$\boxed{C(s) = ① + ②}$

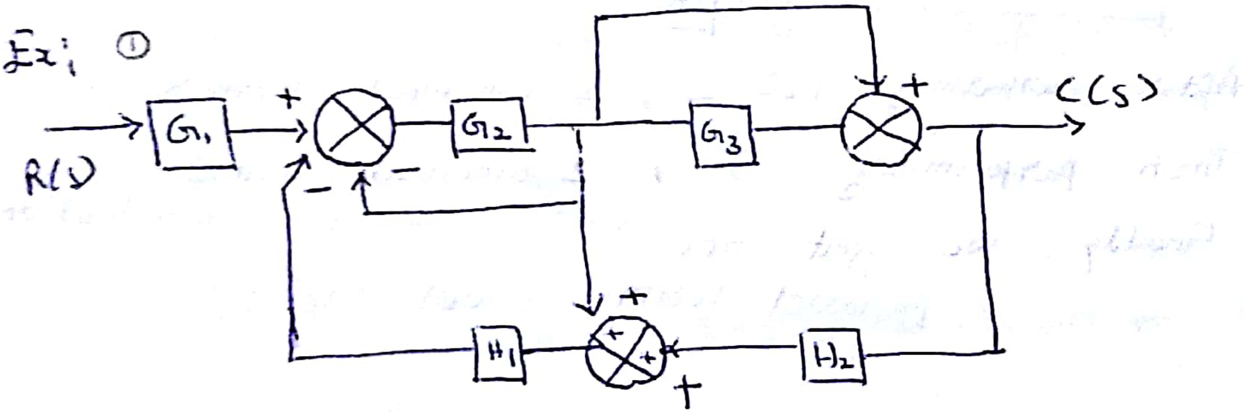
$R_2(s) = 0$

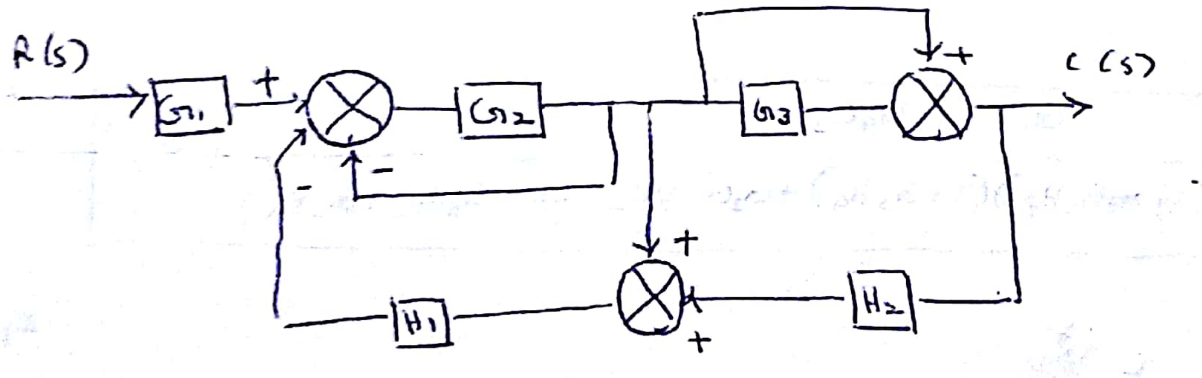
$\frac{C(s)}{R_1(s)} = ?$ — ①

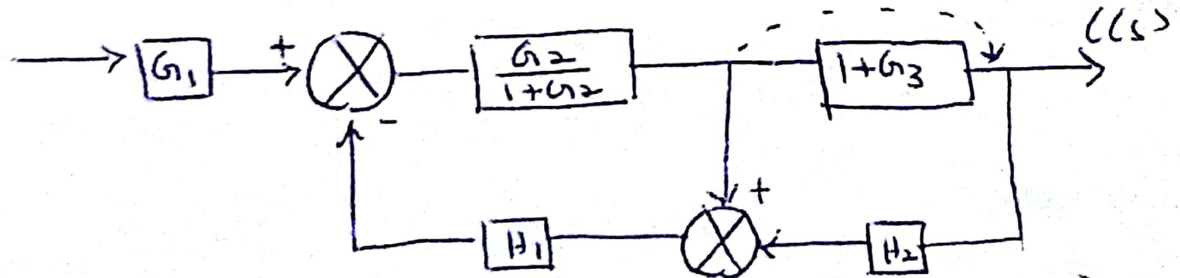
$R_1(s) = 0$

$\frac{C(s)}{R_2(s)} = ?$ — ②

Ex: ①





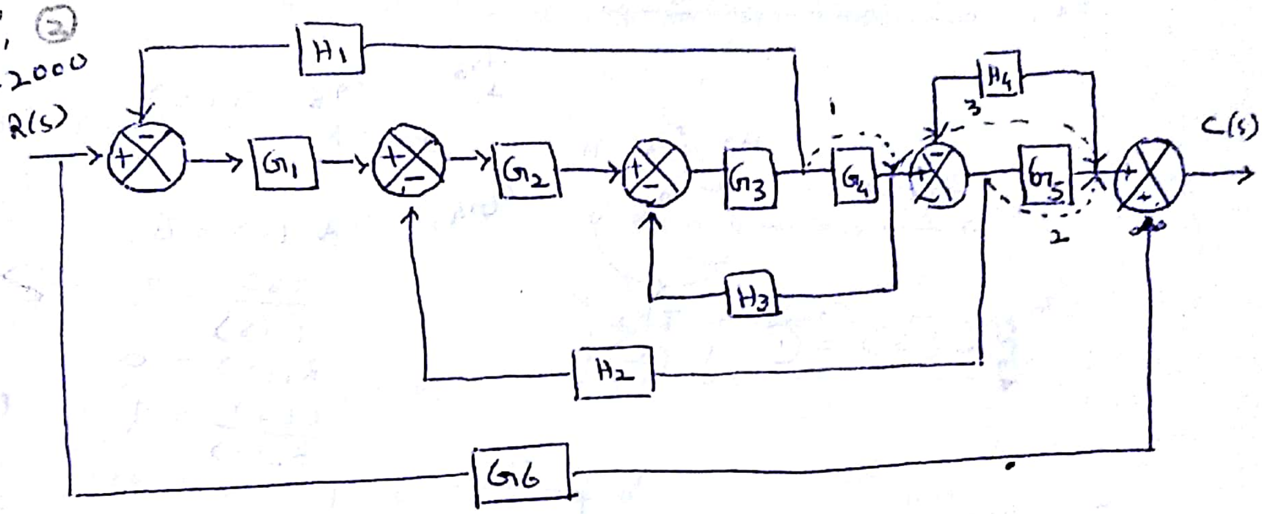


Shift Take off- point after a block (i.e. $\frac{1}{1+G_3}$) then parallel & canonical form & then obtain the final T.F. as

$$R(s) \longrightarrow \boxed{\frac{G_1G_2(1+G_3)}{1+G_2+G_2H_1(H_2+H_2G_3+1)}} \longrightarrow C(s)$$

(Ans.)

Ex: (2)
W-2000



After performing 1 & 2, 2 canonical forms

Then performing 3, 2 canonical forms
(2nd —"— —"— dependent on)

Finally, we get one parallel form. (with G6)

$$R(s) \longrightarrow \boxed{\frac{G_1G_2G_3G_4G_5}{(1+G_3G_4H_3)(1+G_5H_4)+G_2G_3G_4H_2+G_1G_2G_3H_1(1+G_5H_4)} + G_6} \longrightarrow C(s)$$

(Ans.:-)

Check it

Laplace Transforms: Let $f(t)$ be a f^n of "t" defined for all +ve values of "t". Then the Laplace transforms of $f(t)$, denoted by $L[f(t)]$ is defined by

$$L\{f(t)\} = \int_0^\infty e^{-st} f(t)\,dt = F(s)$$

provided that the integral exists, "s" is a parameter which may be real or complex number.

$f(t)$ is called the inverse Laplace Transform of $F(s)$.

Properties: ①: $L\,af(t) = a\,L\,f(t)$

② $L[f(t) + g(t)] = Lf(t) + Lg(t)$

③ $\boxed{1^{st}}$ shifting property

$$L[e^{at} f(t)] = F(s-a)$$

Transform of Derivative:-

④ $L[f'(t)] = sF(s) - f(0)$

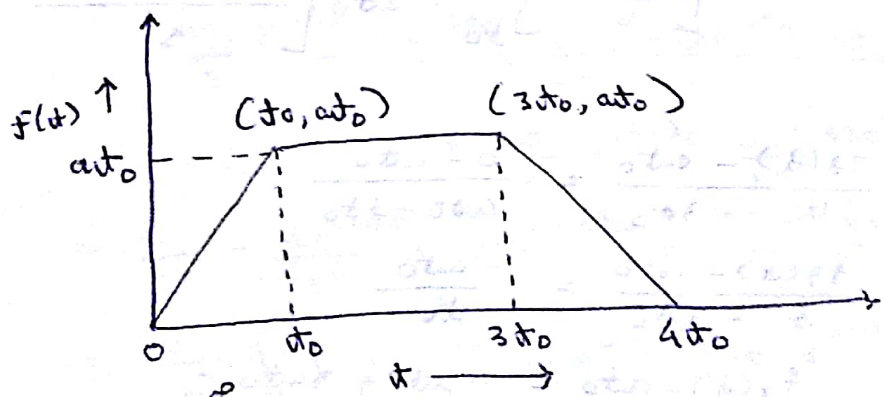
$$L[f''(t)] = s^2F(s) - sf(0) - f(0)$$

Transform of Integral:

$$L\left\{\int_0^t f(u)\,du\right\} = \frac{1}{s}F(s)$$

S-95/S-97

Ex: Find the Laplace transformation for the f^n shown in fig.



$$L[f(t)] = \int_0^\infty e^{-st} f(t)\,dt$$

$$= \int_0^{t_0} e^{-st} f_1(t)\,dt + \int_{t_0}^{3t_0} e^{-st} f_2(t)\,dt + \int_{3t_0}^{4t_0} e^{-st} f_3(t)\,dt + \int_{4t_0}^{\infty} e^{-st} F_4(t)\,dt$$

$$F(s) = F_1(s) + F_2(s) + F_3(s) + F_4(s)$$

$f_1(t)$: Eq^n of straight line

$$\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$$

$$\frac{f_1(t)-0}{t-0} = \frac{at_0-0}{t_0-0}$$

$$\Rightarrow f_1(t) = at$$

$$F_1(s) = \int_0^{t_0} e^{-st} at\, dt = a\int_0^{t_0} te^{-st} dt = a\left[t\int_0^{t_0} e^{-st} dt - \int_0^{t_0} \frac{d}{dt}t \int e^{-st} dt\, dt\right]$$

$$= \left[-\frac{te^{-st}}{s} - \int \frac{e^{-st}}{-s} dt\right]$$

$$= \left[-\frac{te^{-st}}{s} + \frac{1}{s}\int e^{-st} dt\right]$$

$$= a\left[-\frac{te^{-st}}{s} - \frac{1}{s^2}e^{-st} \cancel{dt}\right]_0^{t_0}$$

$$F_1(s) = a\left[-\frac{t_0 e^{-st_0}}{s} - \frac{e^{-st_0}}{s^2} + \frac{1}{s^2}\right] \quad — (1)$$

$f_2(t)$: $f_2(t) = at_0$

$$F_2(s) = \int_{t_0}^{3t_0} e^{-st} at_0\, dt = at_0 \int_{t_0}^{3t_0} e^{-st} dt$$

$$= at_0\left[-\frac{e^{-st}}{s}\right]_{t_0}^{3t_0} = at_0\left[\frac{e^{-st_0} - e^{-3st_0}}{s}\right] — (2)$$

R

—

$f_3(t)$:

$$\frac{f_3(t) - at_0}{t - 3t_0} = \frac{0 - at_0}{4t_0 - 3t_0}$$

$$\frac{f_3(t) - at_0}{t - 3t_0} = \frac{-a\cancel{t_0}}{\cancel{t_0}}$$

$$f_3(t) - at_0 = -at + 3at_0$$

$$f_3(t) = -at + 4at_0$$

Scanned by CamScanner

$$F_3(s) = \int_{3t_0}^{4t_0} e^{-st}(\cancel{a} - at + 4at_0)\,dt$$

$$= 4at_0 \int_{3t_0}^{4t_0} e^{-st}\,dt - a\int_{3t_0}^{4t_0} e^{-st}(t)\,dt$$

$$= 4at_0\left[-\frac{e^{-st}}{s}\right]_{3t_0}^{4t_0} - a\left[-\frac{te^{-st}}{s} - \frac{e^{-st}}{s^2}\right]_{3t_0}^{4t_0}$$

$$= 4at_0\left[\frac{e^{-3st_0} - e^{-4st_0}}{s}\right]$$

$$- a\left[-\frac{4t_0 e^{-4st_0}}{s} - \frac{e^{-4st_0}}{s^2} + \frac{3t_0 e^{-3st_0}}{s} + \frac{e^{-3st_0}}{s^2}\right]$$

$$= \left[\frac{4at_0}{s} - \frac{3at_0}{s} - \frac{a}{s^2}\right]e^{-3st_0} + \left[-\frac{4at_0}{s} + \frac{4at_0}{s} + \frac{a}{s^2}\right]e^{-4st_0}$$

$$= \left[\frac{at_0}{s} - \frac{a}{s^2}\right]e^{-3st_0} + \frac{a}{s^2}e^{-4st_0} \quad \text{—— (3)}$$

$f_4(t)$: $f_4(t) = 0$

$\therefore F_4(s) = 0$ —— (4)

$$F(s) = F_1(s) + F_2(s) + F_3(s) + F_4(s)$$

$$= -\frac{\cancel{at_0}}{\cancel{s}}e^{-st_0} - \frac{ae^{-st_0}}{s^2} + \frac{a}{s^2} + \cancel{\frac{at_0 e^{-st_0}}{s}} - \cancel{\frac{at_0 e^{-3st_0}}{s}}$$

$$+ \frac{\cancel{at_0}}{\cancel{s}}e^{-3st_0} - \frac{a}{s^2}e^{-3st_0} + \frac{a}{s^2}e^{-4st_0}$$

$$= \frac{a}{s^2} - \frac{ae^{-st_0}}{s^2} - \frac{ae^{-3st_0}}{s^2} + \frac{ae^{-4st_0}}{s^2}$$

$$= \frac{a}{s^2}\left[1 - e^{-st_0} - e^{-3st_0} + e^{-4st_0}\right]$$

(Ans.)

Mathematical Modelling of physical systems:

Physical Systems:

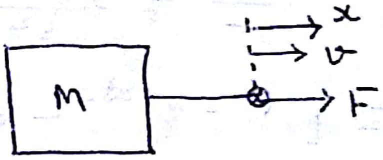
- Electrical System
- Mechanical System
  - Translational Mech. systems
    - Active — Force
    - Passive — M, Spring, damper
    - variables of motion
    - $x(m)$, $\frac{dx}{dt}$ (m/s)
    - & a or $\frac{d^2x}{dt^2}$ (m/s^2)
  - Rotational Mech. systems
    - Active – Torque
    - Passive — J (moment of inertia), spring, damper
    - variable of motion
    - θ (rad), ω or $\frac{d\theta}{dt}$ (rad/sec)
    - α or $\frac{d^2\theta}{dt^2}$ (rad/sec^2)
- Electromechanical System

Mathematical modelling of any physical system is carried out by writing down differential eqns governing that system.

Mathematical modelling of electrical systems is carried out using Kirchoff's law (KCL & KVL) for mech. systems Newton's laws & D'Alemberts principles is used for writing down the differential eqns. Then by taking Laplace Transform of these eqns & with proper algebraic manipulations the reqd. TF is obtained.

(1) Translational Mechanical System:

(1) Mass element



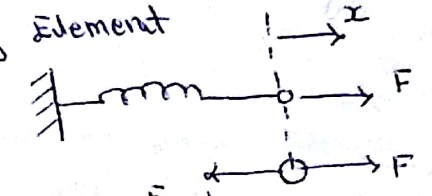
$F_I \longleftarrow\circ\longrightarrow F$

when an external force is applied to the mass element, an inertia force (due to mass of the body) is developed which opposes the motion i.e. applied force. According to Newton's 2nd law

$= M\frac{d^2x}{dt^2}$

$$F_i = M\frac{d^2x}{dt^2}$$

(2) Spring Element

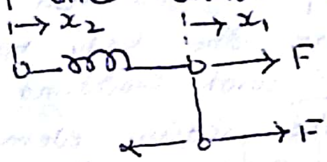


$F_r = kx$

When an external force "F" is applied to the spring element, a restoring force is developed which opposes the motion i.e. applied force. This restoring force is proportional to the displ. & is given by

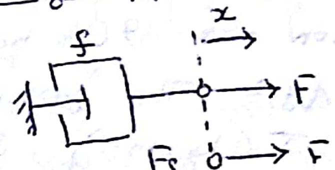
$F_r \propto x \quad \therefore \; F_r = kx$

If both the ends are displaced.



$$F_r = k(x_1 - x_2)$$

(3) Damper element:



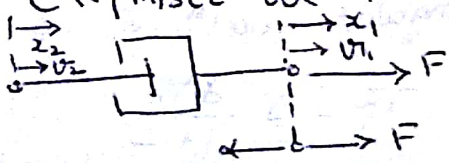
It consist of oil filled cylinder & a piston. The relative motion betn the piston & cylinder is opposed by the oil having coeff. of viscous friction 'f'.

$$F_f \propto \frac{dx}{dt} \qquad F_f = f\frac{dx}{dt}$$

When an external force is applied to a Damper element, a viscous frictional force ~~f~~ (damping force) is developed which opposes the motion i.e. applied force. This is proportional to the velocity.

$$F_f = f\frac{dx}{dt}$$

where "f" is the coeff. of viscous friction (N/m/sec or N-sec/m)

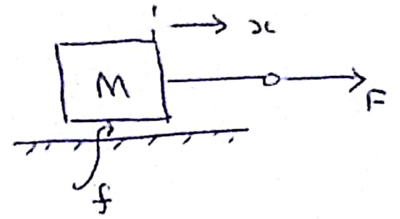


$x_1 > x_2$

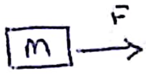
$$F_f = f\left(\frac{dx_1}{dt} - \frac{dx_2}{dt}\right)$$

Scanned by CamScanner

m



$F_f = f \frac{dx}{dt}$



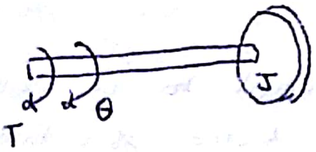
$M\frac{d^2x}{dt^2} \leftarrow$

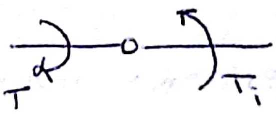
$f\frac{dx}{dt} \leftarrow$

D'Alembert's principle for translational motion:

Summation of forces applied on a mass element is equal to the summation of forces opposing the motion.

Rotational mechanical systems: The mechanical systems having —"— motion i.e. where the body moves / rotates around a fixed axis & with constant distance from that axis. Torque is the active element & the variables of motion are θ (radians), $\omega = \frac{d\theta}{dt}$ (rad/sec) & $\alpha = d^2\theta/dt^2$. The passive elements are inertia element J (kg-m²), torsional spring element (~~k sti~~ (having stiffness k) & damper ~~eleme~~ —"— (having viscous friction coeff. 'f').



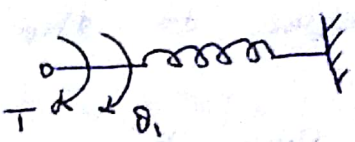


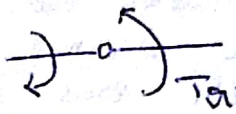
$T_i = J\frac{d^2\theta}{dt^2}$

When an external torque is applied to an inertia element having moment of inertia J, an —"— torque is developed due to inertia of the body, which opposes the motion i.e. applied torque. Acc. to Newton 2nd law this inertia torque is proportional to the angular accⁿ & is given by

$$T_i \propto \frac{d^2\theta}{dt^2} \qquad T_i = J\frac{d^2\theta}{dt^2}$$

Torsional spring element :

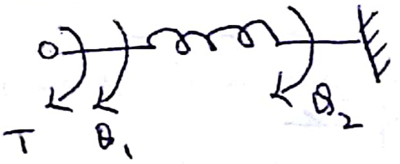




$T_r = K\theta$

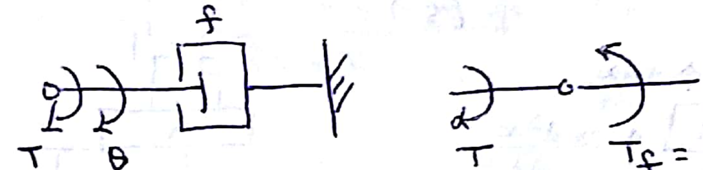
When an external torque is applied to a spring element having stiffness K, a restoring torque is developed which opposes the motion i.e. applied —"—. This T_r is given as $T_r \propto \theta$ $\quad T_r = K\theta$ where K is spring constant / stiffness of the spring (N-m/rad)

When both the ends are displaced



$\theta_1 > \theta_2$ $\quad T_r = k(\theta_1 - \theta_2)$

Damper element :



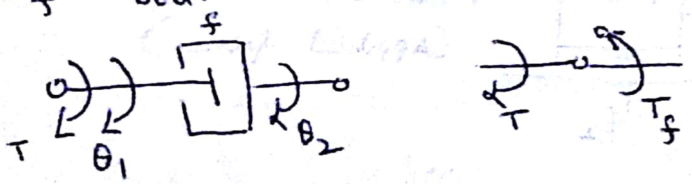
$T_f = f\dfrac{d\theta}{dt}$

When external torque is applied to a damper element, a viscous frictional torque / damping torque is developed which opposes the motion i.e. applied torque. This is proportional to the angular velocity & is given by $T_f \propto \dfrac{d\theta}{dt}$

$$T_f = f\frac{d\theta}{dt}$$

where f is viscous friction coeff. (N-m/rad/sec or $\dfrac{\text{N-m-sec}}{\text{rad}}$)

If both the ends are displaced



$T_f = f\left(\dfrac{d\theta_1}{dt} - \dfrac{d\theta_2}{dt}\right)$

or $\cancel{f(\dot\theta_1 - \dot\theta_2)}$

$f(\dot\theta_1 - \dot\theta_2)$

D'Alembert's principle for rotational element mech. system :- Summation of all the applied torques on the inertia element is equal to the summation of torques opposing the motion.

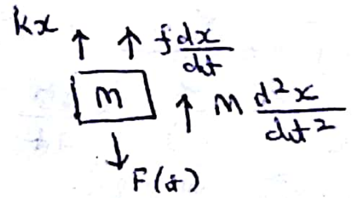
Procedure: ① Show the variables i.e. displs on Mass/Inertia element in their proper dirns.

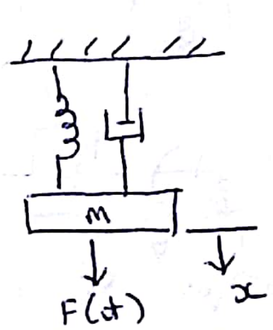
② Draw FBD on each mass/inertia element showing various forces/torques. Draw FBD on every mass element.

③ Apply D'Alembert's principle at the mass/inertia element & write down the differential eqns.

④ Take Laplace Transform of the eqns obtd. in ~~③~~ ③ & with proper algebraic manipulations obtain the reqd. T.F.

Ex: T.F. $= \dfrac{X(s)}{F(s)}$





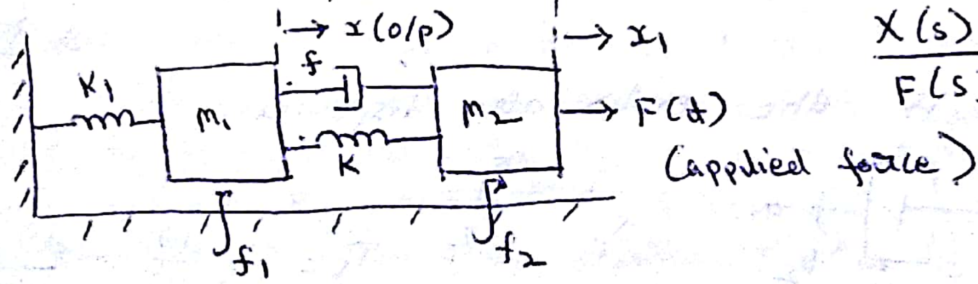
$$F(t) = M\frac{d^2x}{dt^2} + f\frac{dx}{dt} + kx$$

Taking L.T.

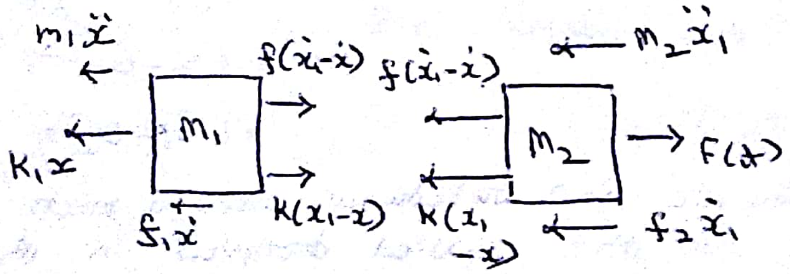
$$F(s) = Ms^2X(s) + fsX(s) + kX(s)$$
$$= X(s)(Ms^2 + fs + k)$$

$$T.F. = \frac{X(s)}{F(s)} = \frac{1}{Ms^2+fs+k}$$

Ex:



$\dfrac{X(s)}{F(s)} = ?$



From FBD of m_2

$$F(t) = m_2\ddot{x}_1 + f(\dot{x}_1 - \dot{x}) + f_2\dot{x}_1 + K(x_1 - x)$$

Taking Laplace Transform (at zero initial condn)

$$F(s) = m_2 s^2 X_1(s) + fsX_1(s) - fsX(s) + f_2 sX_1(s) + KX_1(s) - KX(s)$$

$$F(s) = X_1(s)[m_2 s^2 + fs + f_2 s + K] - X(s)[fs + K] \quad \text{— (1)}$$

From FBD of m_1

$$f(\dot{x}_1 - \dot{x}) + K(x_1 - x) = m_1\ddot{x} + f_1\dot{x} + K_1 x$$

$$fsX_1(s) - fsX(s) + KX_1(s) - KX(s) = m_1 s^2 X(s) + f_1 sX(s) + K_1 X(s)$$

$$X_1(s)(fs + K) = X(s)[m_1 s^2 + f_1 s + K_1 + fs + K]$$

$$X_1(s) = \frac{X(s)[m_1 s^2 + f_1 s + K_1 + fs + K]}{fs + K} \quad \text{— (2)}$$

(2) in (1)

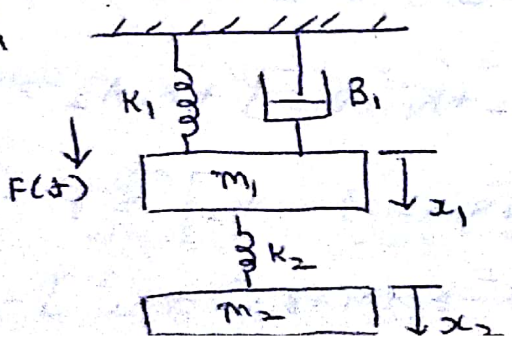
$$F(s) = \frac{X(s)[m_1 s^2 + f_1 s + K_1 + fs + K]}{fs + K}[m_2 s^2 + fs + f_2 s + K] - X(s)[fs + K]$$

$$= \frac{X(s)}{fs + K}\left[(m_1 s^2 + f_1 s + K_1 + fs + K)(m_2 s^2 + fs + f_2 s + K) - (fs + K)^2\right]$$

$$\boxed{\frac{X(s)}{F(s)} = \frac{fs + K}{[m_1 s^2 + (f + f_1)s + K_1 + K][m_2 s^2 + (f + f_2)s + K] - [fs + K]^2}}$$

(Ans.)

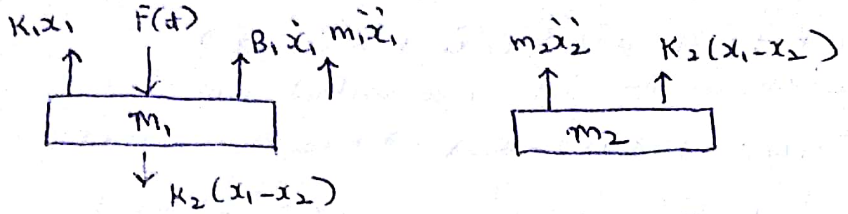
Ex:



$$\frac{X_2(s)}{F(s)} = ? \quad \frac{X_1(s)}{X_2(s)} = ?$$

$$\frac{X_1(s)}{F(s)} = ?$$

[Only system Equations]



$$m_1\ddot{x}_1 = K_2(x_1-x_2) + F(t) - K_1x_1 - B_1\dot{x}_1$$

$$m_2\ddot{x}_2 = -K_2(x_1-x_2)$$

Taking the Laplace

$$m_1s^2X_1(s) = K_2[X_1(s)-X_2(s)] + F(s) - K_1X_1(s) - B_1sX_1(s) \quad \text{-- (1)}$$

$$m_2s^2X_2(s) = -K_2[X_1(s)-X_2(s)] \quad \text{-- (2)}$$

$$m_2s^2X_2 = -K_2X_1 + K_2X_2 \quad \text{-- (3)}$$

$$m_1s^2X_1 = K_2X_1 - K_2X_2 + F - K_1X_1 - B_1sX_1 \quad \text{.. (4)}$$

(3) $\Rightarrow$ $$X_2 = \frac{-K_2X_1}{m_2s^2 - K_2}$$

(4) $\Rightarrow$ $$m_1s^2X_1 = \frac{K_2^2X_1}{m_2s^2-K_2} + K_2X_1 + F - K_1X_1 - B_1sX_1$$

$$X_1\left[m_1s^2 - \frac{K_2^2}{m_2s^2-K_2} - K_2 + K_1 + B_1s\right] = F \quad \text{— (5)}$$

(3) $\Rightarrow$ $$X_1 = \frac{(m_2s^2-K_2)X_2}{-K_2}$$

(1) $\Rightarrow$ $$X_1[m_1s^2 - K_2 + K_1 + B_1s] + K_2X_2 = F$$

$$-\frac{(m_2s^2-K_2)X_2}{K_2}[m_1s^2 - K_2 + K_1 + B_1s] + K_2X_2 = F$$

$$X_2\left[-\frac{(m_2s^2-K_2)}{K_2}(m_1s^2 - K_2 + K_1 + B_1s) + K_2\right] = F$$

$$\frac{X_2}{K_2}\left[-(m_2s^2-K_2)(m_1s^2 - K_2 + K_1 + B_1s) + K_2^2\right] = F \quad \text{— (6)}$$

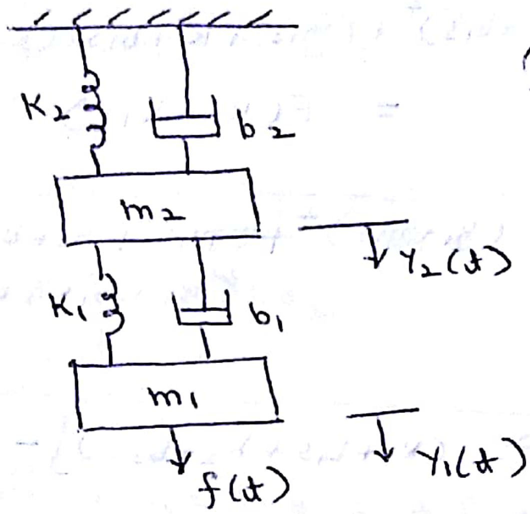
Scanned by CamScanner

$$X_1\left[m_1s^2 - \frac{K_2^2}{m_2s^2-K_2} - K_2+K_1+B_1s\right] = \frac{X_2}{K_2}\left[-(m_2s^2-K_2)(m_1s^2-K_2+K_1+B_1s) + K_2^2\right]$$

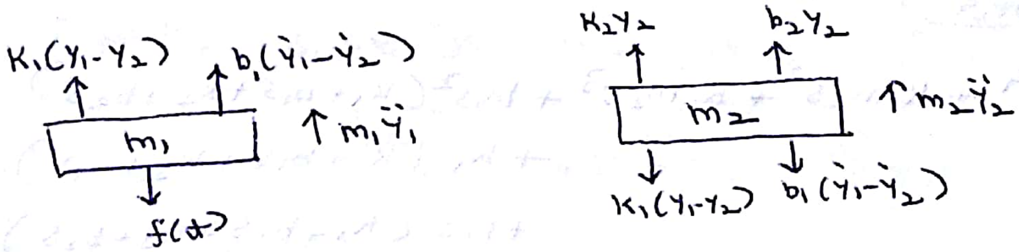
$$\frac{X_1}{X_2} = \frac{\frac{1}{K_2}\left[-(m_2s^2-K_2)(m_1s^2-K_2+K_1+B_1s)+K_2^2\right]}{m_1s^2 - \frac{K_2^2}{m_2s^2-K_2} - K_2+K_1+B_1s}$$

(Ans.)

Ex:



Calculate $\frac{Y_2}{F} = ?$



$$m_1\ddot{y}_1 = f(t) - K_1(Y_1-Y_2) - b_1(\dot{y}_1-\dot{y}_2)$$

$$m_2\ddot{Y}_2 = K_1(Y_1-Y_2) + b_1(\dot{Y}_1-\dot{Y}_2) - K_2Y_2 - b_2\dot{Y}_2$$

Taking Laplace Transform

$$m_1s^2Y_1 = F - K_1(Y_1-Y_2) - b_1s(Y_1-Y_2) \quad \text{--- (1)}$$

$$m_2s^2Y_2 = K_1(Y_1-Y_2) + b_1s(Y_1-Y_2) - K_2Y_2 - b_2sY_2 \quad \text{--- (2)}$$

(1) => $m_1s^2Y_1 = F - K_1Y_1 + K_1Y_2 - b_1sY_1 + b_1sY_2$

$$Y_1[m_1s^2+K_1+b_1s] = F + K_1Y_2 + b_1sY_2$$

$$Y_1 = \frac{F + Y_2(K_1+b_1s)}{m_1s^2+K_1+b_1s}$$

(2) =>

$$m_2 s^2 Y_2 = Y_1[K_1 + b_1 s] - K_1 Y_2 - b_1 s Y_2 - K_2 Y_2 - b_2 s Y_2$$

$$m_2 s^2 Y_2 = \frac{F + Y_2(K_1 + b_1 s)}{m_1 s^2 + K_1 + b_1 s}[K_1 + b_1 s] - Y_2[K_1 + b_1 s + K_2 + b_2 s]$$

$$m_2 s^2 Y_2 (m_1 s^2 + K_1 + b_1 s) = [F(K_1 + b_1 s) + Y_2 (K_1 + b_1 s)^2] - Y_2 (m_1 s^2 + K_1 + b_1 s)(K_1 + b_1 s + K_2 + b_2 s)$$

$$Y_2\left[m_2 s^2 (m_1 s^2 + K_1 + b_1 s) - (K_1 + b_1 s)^2 + (m_1 s^2 + K_1 + b_1 s)(K_1 + b_1 s + K_2 + b_2 s)\right] = F(K_1 + b_1 s)$$

$$\frac{Y_2}{F} = \frac{K_1 + b_1 s}{m_2 s^2 (m_1 s^2 + K_1 + b_1 s) - (K_1 + b_1 s)^2 + (m_1 s^2 + K_1 + b_1 s)(K_1 + b_1 s + K_2 + b_2 s)}$$

$$= \frac{K_1 + b_1 s}{(m_1 s^2 + K_1 + b_1 s)[m_2 s^2 + (K_1 + b_1 s + K_2 + b_2 s)] - (K_1 + b_1 s)^2}$$

Dinominator

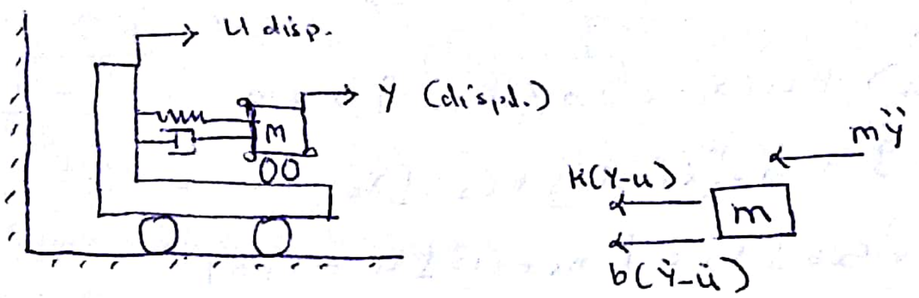
$$\Rightarrow m_1 m_2 s^4 + K_1 m_2 s^2 + b_1 m_2 s^3 + m_1 s^2 (K_1 + b_1 s + K_2 + b_2 s) + K_1 (K_1 + b_1 s + K_2 + b_2 s) + b_1 s (K_1 + b_1 s + K_2 + b_2 s) - K_1^2 - b_1^2 s^2 - 2K_1 b_1 s$$

$$\Rightarrow m_1 m_2 s^4 + [m_1 b_1 + b_1 m_2 + m_1 b_2] s^3 + [K_1 m_2 + K_1 m_1 + K_2 m_1 + \cancel{b_1^2} + b_1 b_2 - \cancel{b_1^2}] s^2 + [K_1 B_2 + \cancel{K_1 B_1} + K_2 B_1 + \cancel{K_1 B_1} - \cancel{2K_1 B_1}] s + [\cancel{K_1^2} - \cancel{K_1^2} + K_1 K_2]$$

$$\frac{Y_2}{F} = \frac{K_1 + b_1 s}{m_1 m_2 s^4 + [(b_1 + b_2) m_1 + b_1 m_2] s^3 + [K_1 m_2 + K_1 m_1 + K_2 m_1 + b_1 b_2] s^2 + [K_1 B_2 + K_2 B_1] s + K_1 K_2}$$

(Ans.)

Ex:

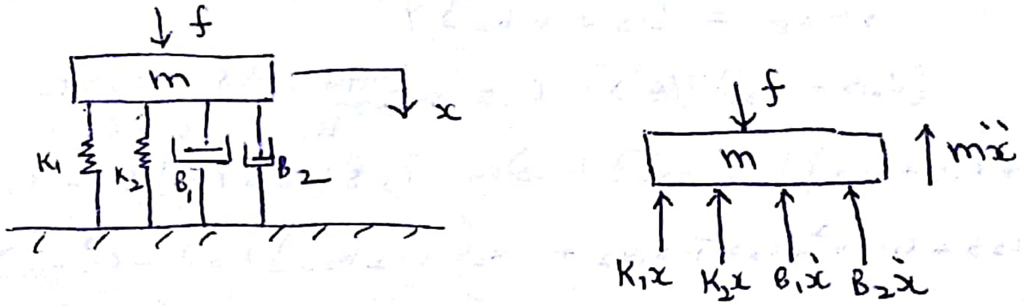


$$m\ddot{y} + b(\dot{y}-\dot{u}) + k(y-u) = 0$$

$$(ms^2 + bs + k)Y(s) = (bs+k)U(s)$$

$$\frac{Y(s)}{U(s)} = \frac{bs+k}{ms^2+bs+k} \quad \text{(Ans.)}$$

Ex:



$$m\ddot{x} + K_1x + K_2x + B_1\dot{x} + B_2\dot{x} = f$$

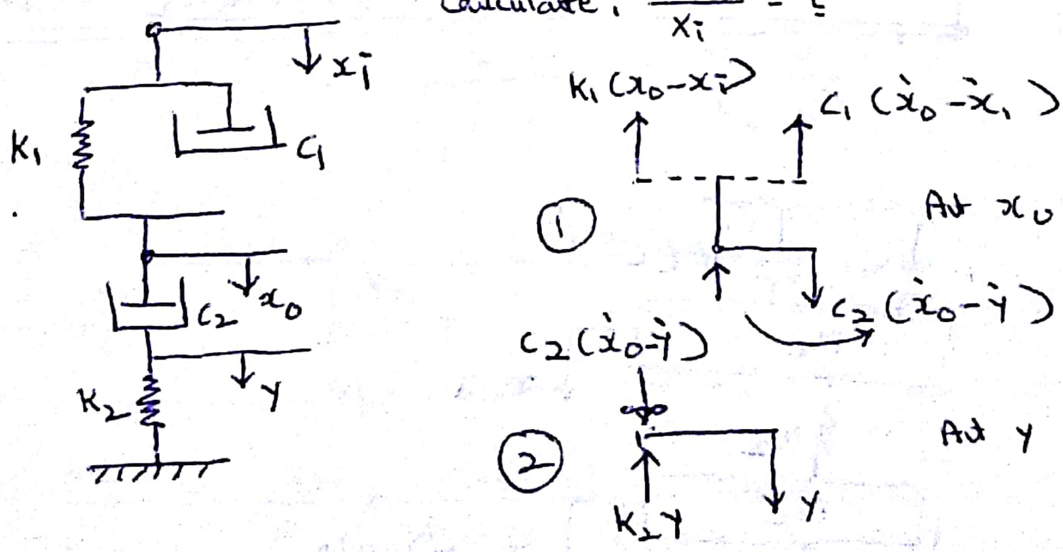
$$[ms^2 + (B_1+B_2)s + K_1+K_2]X(s) = F(s)$$

$$\frac{X(s)}{F(s)} = \frac{1}{ms^2+(B_1+B_2)s+K_1+K_2}$$

W-97

Ex:

Calculate: $\frac{Y}{X_i} = ?$



At x_0 ;

$$K_1(x_0 - x_i) + C_1(\dot{x}_0 - \dot{x}_i) + C_2(\dot{x}_0 - \dot{y}) = 0$$

$$K_1[X_0 - X_i] + C_1 s[X_0 - X_i] + C_2 s[X_0 - Y] = 0$$

$$[K_1 + C_1 s + C_2 s]X_0 = [K_1 + C_1 s]X_i + C_2 s Y$$

$$X_0 = \frac{[K_1 + C_1 s]X_i + C_2 s Y}{K_1 + C_1 s + C_2 s}$$

At Y; $C_2(\dot{x}_0 - \dot{y}) = K_2 Y$

$$C_2 s[X_0 - Y] = K_2 Y$$

$$C_2 s X_0 = [C_2 s + K_2]Y$$

$$[C_2 s + K_2]Y(s) = C_2 s \times \frac{[K_1 + C_1 s]X_i + C_2 s Y}{K_1 + C_1 s + C_2 s}$$

$$[K_1 + C_1 s + C_2 s][C_2 s + K_2]Y(s) = C_2 s[K_1 + C_1 s]X_i + C_2^2 s^2 Y$$

$$[K_1 C_2 s + C_1 C_2 s^2 + C_2 s^2 + K_1 K_2 + C_1 K_2 s + C_2 K_2 s]Y(s) - C_2^2 s^2 Y$$

$$= C_2 s[K_1 + C_1 s]X_i$$

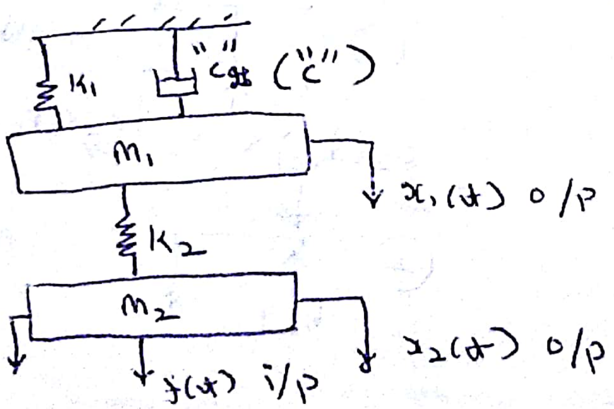
$$[K_1 C_2 s + C_1 C_2 s^2 + C_2 s^2 + K_1 K_2 + C_1 K_2 s + C_2 K_2 s - C_2^2 s^2]Y$$

$$\Rightarrow \cancel{K_1 s C_2} \; [K_1 C_2 s + C_1 C_2 s^2]X_i$$

$$\boxed{\frac{Y}{X_i} = \frac{K_1 C_2 s + C_1 C_2 s^2}{K_1 C_2 s + C_1 C_2 s^2 + C_2 s^2 + K_1 K_2 + C_1 K_2 s + C_2 K_2 s - C_2^2 s^2}}$$

(Ans.)

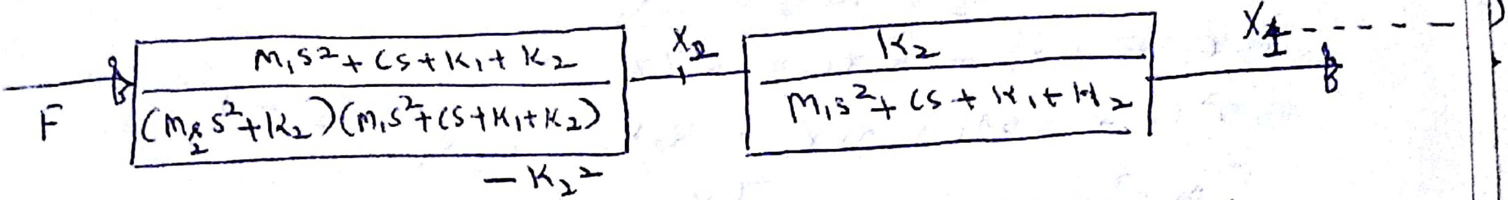
W-98
Ex';



Scanned by CamScanner

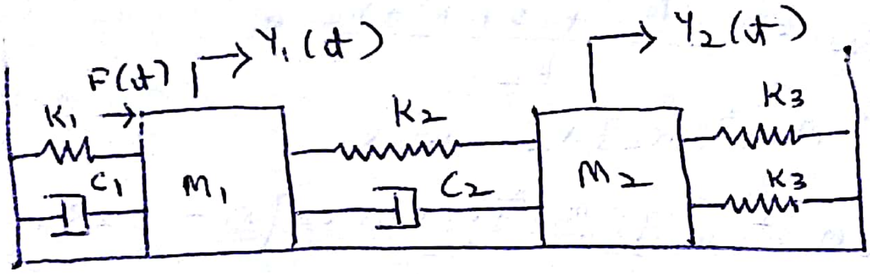
$$\frac{X_2}{F} = \frac{K_2}{(m_2 s^2 + K_2)(m_1 s^2 + cs + K_1 + K_2) - K_2^2} \times \frac{m_1 s^2 + cs + K_1 + K_2}{K_2}$$

$$\frac{X_2}{F} = \frac{m_1 s^2 + cs + K_1 + K_2}{(m_2 s^2 + K_2)(m_1 s^2 + cs + K_1 + K_2) - K_2^2}$$

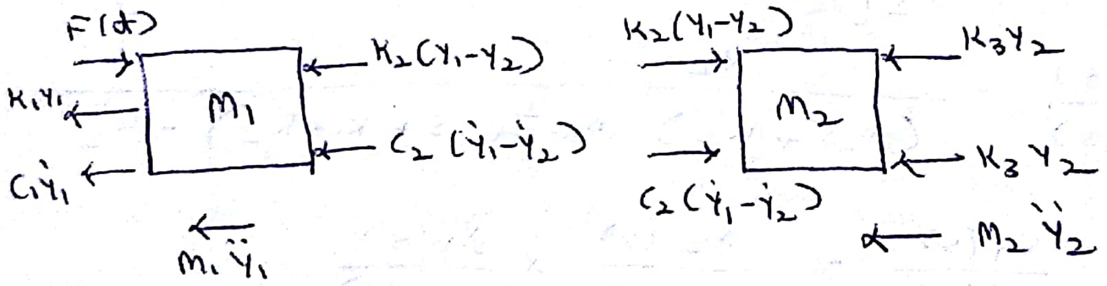


(Ans: -o is valid)

Ex: S-99



Find $\frac{Y_1(s)}{F(s)}$



$$F(t) = K_1 Y_1 + K_2(Y_1 - Y_2) + C_1 \dot{Y}_1 + C_2(\dot{Y}_1 - \dot{Y}_2) + m_1 \ddot{Y}_1$$

$$F(s) = K_1 Y_1 + K_2(Y_1 - Y_2) + C_1 s Y_1 + C_2 s(Y_1 - Y_2) + m_1 s^2 Y_1$$

$$F(s) = [K_1 + K_2 + C_1 s + C_2 s + m_1 s^2] Y_1 - [K_2 + C_2 s] Y_2 \quad \text{--- (1)}$$

$$K_2(Y_1 - Y_2) + C_2(\dot{Y}_1 - \dot{Y}_2) = K_3 Y_2 + K_3 Y_2 + M_2 \ddot{Y}_2$$

$$K_2(Y_1 - Y_2) + C_2 s(Y_1 - Y_2) = 2K_3 Y_2 + M_2 s^2 Y_2$$

$$[K_2 + C_2 s] Y_1 = \cancel{K Y_2}$$
$$= [K_2 + C_2 s + 2K_3 + m_2 s^2] Y_2$$

$$Y_2 = \frac{K_2 + C_2 s}{K_2 + C_2 s + 2K_3 + m_2 s^2} Y_1 \quad --- \quad (2)$$

$$F(s) = [K_1 + K_2 + C_1 s + C_2 s + m_1 s^2] Y_1 - \frac{[K_2 + C_2 s]^2}{K_2 + C_2 s + 2K_3 + m_2 s^2} Y_1$$

$$F(s) = \left[(K_1 + K_2 + C_1 s + C_2 s + m_1 s^2) - \frac{(K_2 + C_2 s)^2}{K_2 + C_2 s + 2K_3 + m_2 s^2} \right] Y_1$$

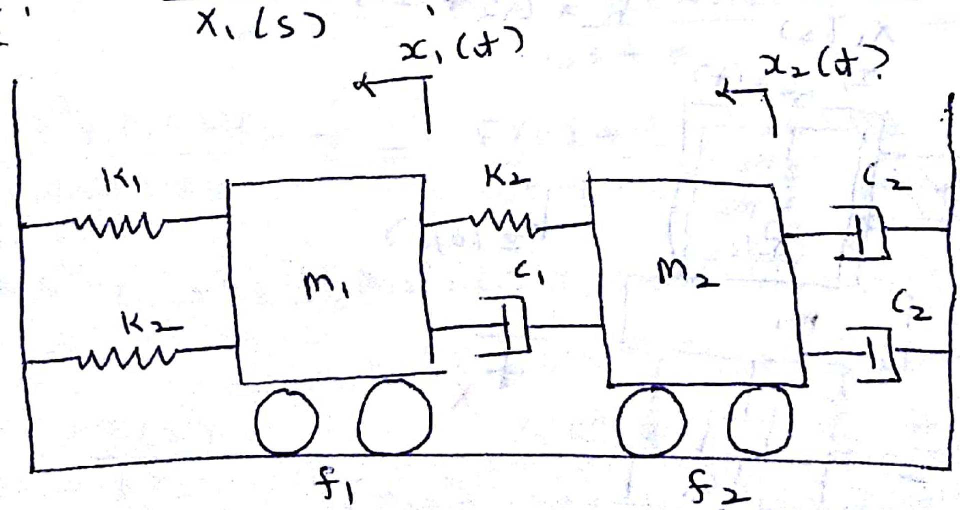
$$F(s) = \frac{1}{(K_2 + C_2 s + 2K_3 + m_2 s^2)} \left[(K_2 + C_2 s + 2K_3 + m_2 s^2)(K_1 + K_2 + C_1 s + C_2 s + m_1 s^2) - (K_2 + C_2 s)^2 \right] Y_1$$

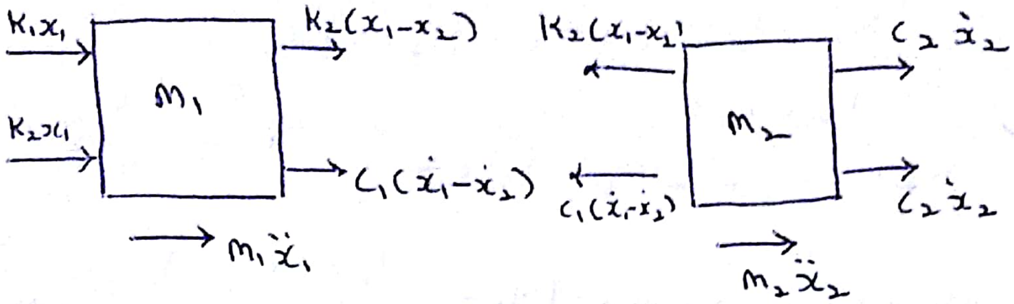
$$\frac{Y_1(s)}{F(s)} = \frac{K_2 + C_2 s + 2K_3 + m_2 s^2}{[K_2 + C_2 s + 2K_3 + m_2 s^2][K_1 + K_2 + C_1 s + C_2 s + m_1 s^2] - (K_2 + C_2 s)^2}$$

(Ans.)

W-2000

Ex: $\frac{X_2(s)}{X_1(s)} = ?$





$$m_1\ddot{x}_1 + k_1x_1 + k_2x_1 + k_2(x_1-x_2) + c_1(\dot{x}_1-\dot{x}_2) = 0$$

$$m_1 s^2 X_1 + k_1X_1 + k_2X_1 + k_2X_1 - k_2X_2 + c_1 sX_1 - c_1 sX_2 = 0$$

$$\cancel{[m_1 s^2 + (k_1 + 2k_2)X_1}$$

$$[m_1 s^2 + k_1 + 2k_2 + c_1 s]X_1 - (c_1 s + k_2]X_2 = 0$$

$$\frac{X_2(s)}{X_1(s)} = \frac{m_1 s^2 + k_1 + 2k_2 + c_1 s}{c_1 s + k_2}$$

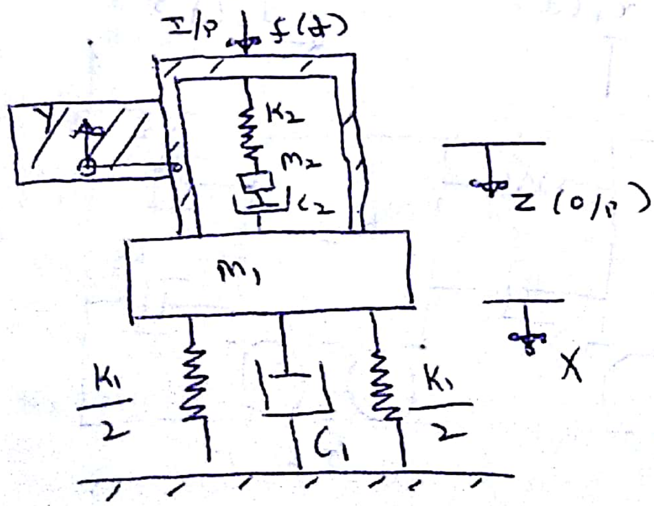
$$k_2(x_1-x_2) + c_1(\dot{x}_1-\dot{x}_2) = 2c_2\dot{x}_2 + m_2\ddot{x}_2$$

$$k_2[X_1-X_2] + c_1 s[X_1-X_2] = 2c_2 sX_2 + m_2 s^2 X_2$$

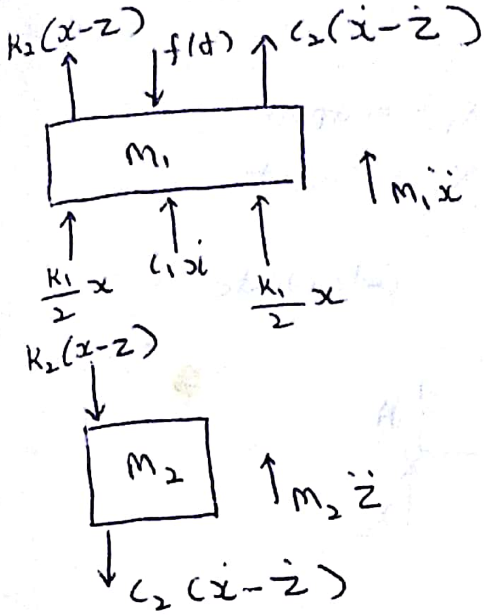
$$[k_2 + c_1 s]X_1 = [k_2 + c_1 s + 2c_2 s + m_2 s^2]X_2$$

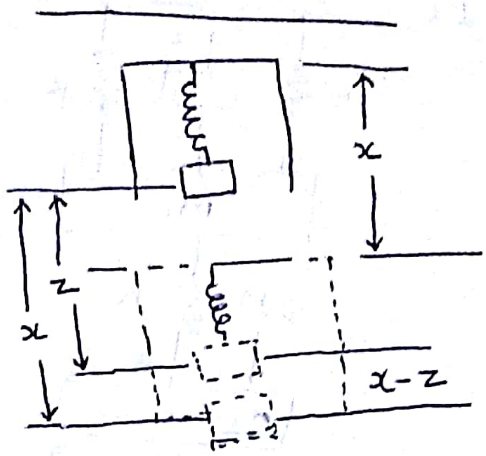
$$\frac{X_2(s)}{X_1(s)} = \frac{k_2 + c_1 s}{k_2 + c_1 s + 2c_2 s + m_2 s^2}$$

Ex:



S-99 Ex' $Y_1(S)$ 9





$$f(t) = m_1\ddot{x} + c_1\dot{x} + c_2(\dot{x}-\dot{z}) + \frac{k_1}{2}x + \frac{k_1}{2}x + k_2(x-z)$$

$$F(s) = m_1 s^2 X + c_1 s X + c_2 s(X-Z) + k_1 X + k_2(X-Z)$$

$$F(s) = [m_1 s^2 + c_1 s + c_2 s + k_1 + k_2]X - [c_2 s + k_2]Z$$

$$X(s) = \frac{F(s) + [c_2 s + k_2]Z}{m_1 s^2 + c_1 s + c_2 s + k_1 + k_2} \quad \text{——} \quad (1)$$

$$m_2\ddot{z} - k_2(x-z) - c_2(\dot{x}-\dot{z}) = 0$$

$$m_2 s^2 Z - k_2[X-Z] - c_2 s[X-\cancel{Z}] = \cancel{0}$$

$$[m_2 s^2 + k_2 + c_2 s]Z = [k_2 + c_2 s]X$$

$$X(s) = \frac{m_2 s^2 + c_2 s + k_2}{c_2 s + k_2} Z \quad \text{——} \quad (2)$$

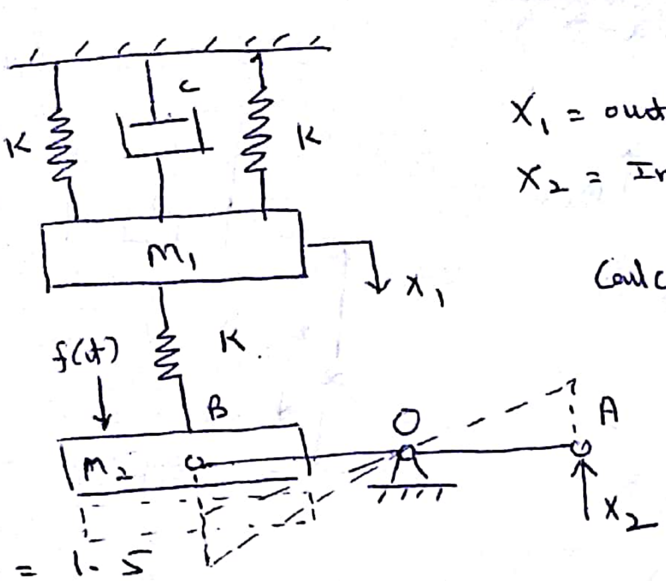
$$\frac{m_2 s^2 + c_2 s + k_2}{c_2 s + k_2} Z = \frac{F(s) + [c_2 s + k_2]Z}{m_1 s^2 + c_1 s + c_2 s + k_1 + k_2}$$

$$[m_2 s^2 + c_2 s + k_2][m_1 s^2 + c_1 s + c_2 s + k_1 + k_2] Z(s)$$
$$= [c_2 s + k_2]F(s) + [c_2 s + k_2]^2 Z(s)$$

$$\frac{Z(s)}{F(s)} = \frac{c_2 s + k_2}{[m_2 s^2 + c_2 s + k_2][m_1 s^2 + c_1 s + c_2 s + k_1 + k_2] - [c_2 s + k_2]^2}$$

(Ans.)

S-98 Ex:



X_1 = output

X_2 = Input

Calculate $\frac{X_1(s)}{X_2(s)} = ?$

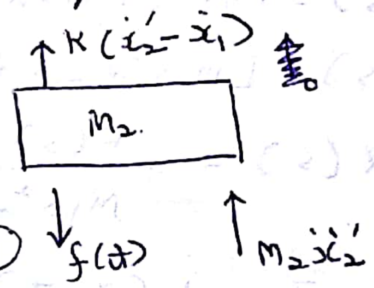
$\frac{OB}{OA} = 1.5$

$\cancel{x_2 \times OA = x_2' \times OB}$

$\frac{OB}{x_2'} = \frac{OA}{x_2}$ $\quad \cancel{x_2' = x_2 \times \frac{OA}{OB} = \frac{1}{1.5} x_2}$

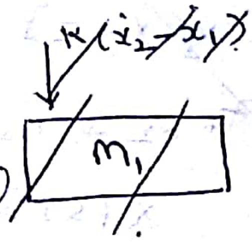
$x_2' = \cancel{\frac{OA}{}} \frac{OB}{OA} x_2$

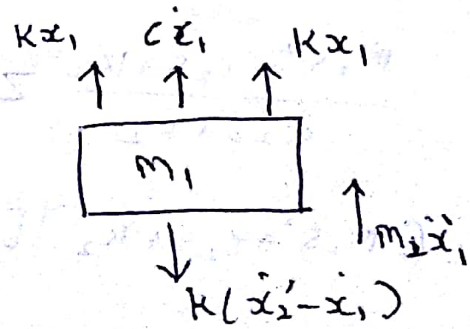
$= 1.5 x_2$



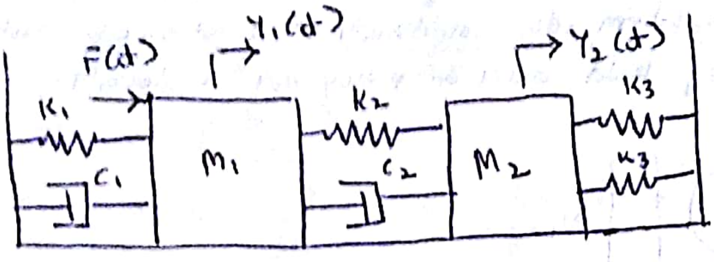
$f(t) = M_2 \ddot{x}_2' + K(x_2' - x_1)$ — (1)

$K(x_2' - x_1) = 2Kx_1 + C\dot{x}_1$
$+ K(x_2' - x_1)$ — (2)

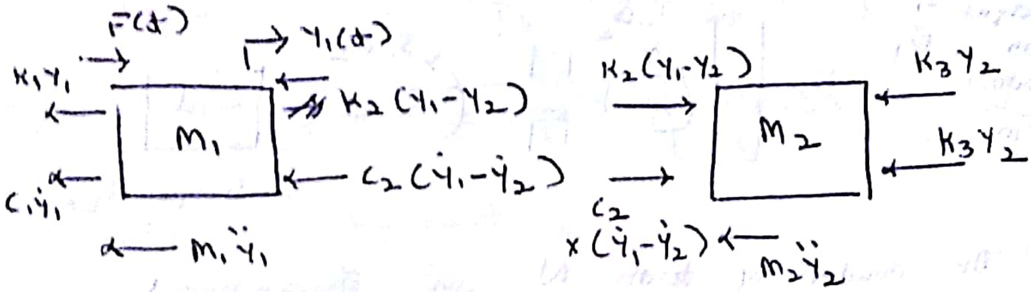




S-99 Ex: $\frac{Y_1(s)}{F(s)} = ?$



Repeat



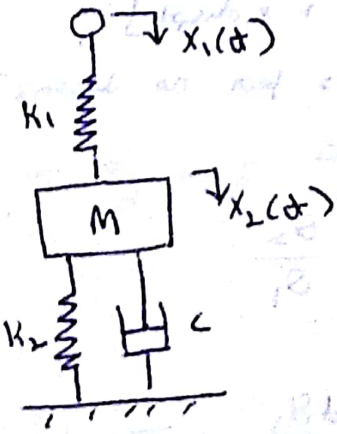
$$F(t) = K_1Y_1 + K_2(Y_1-Y_2) + C_1\dot{Y}_1 + C_2(\dot{Y}_1-\dot{Y}_2) + M_1\ddot{Y}_1 \quad \text{—} \ (1)$$

$$F(s) = K_1Y_1 + K_2(Y_1-Y_2) + C_1sY_1 + C_2s(Y_1-Y_2) + M_1s^2Y_1 \quad \text{—} \ (3)$$

$$K_2(Y_1-Y_2) + C_2(\dot{Y}_1-\dot{Y}_2) = K_3Y_2 + K_3Y_2 + M_2\ddot{Y}_2 \quad \text{—} \ (2)$$

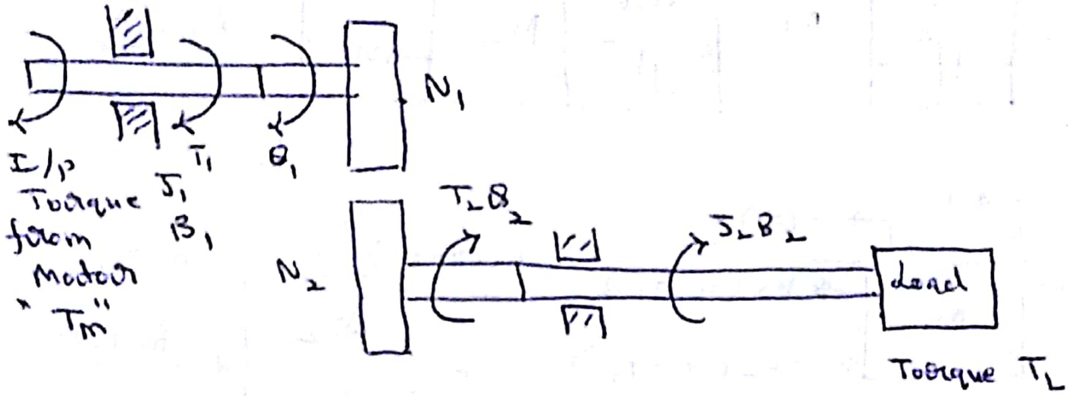
$$K_2(Y_1-Y_2) + C_2s(Y_1-Y_2) = 2K_3Y_2 + M_2s^2Y_2 \quad \text{—} \ (4)$$

S-99 H.W. Ex:

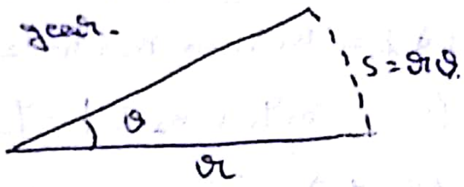


$\frac{X_2(s)}{X_1(s)} = ?$

Gear Trains :- Mech. energy can be transmitted from one part of a system to another via devices such as gear trains, timing belt over a pulley or a lever.



(1) The number of teeth N are proportional to the radius "r" of a gear.



$$r \propto N$$

$$\therefore \frac{r_1}{r_2} = \frac{N_1}{N_2}$$

$$r_1 N_2 = r_2 N_1$$

(2) $$s = r_1\theta_1 = r_2\theta_2$$

$$\therefore r_1\theta_1 = r_2\theta_2$$

(3) $$W.D. = T.\theta \quad [W = F \times displ]$$

$$T_1\theta_1 = T_2\theta_2 \quad [\text{This is for no losses}]$$

$$\frac{r_1}{r_2} = \frac{\theta_2}{\theta_1} \qquad \frac{T_1}{T_2} = \frac{\theta_2}{\theta_1}$$

$$\frac{T_1}{T_2} = \frac{r_1}{r_2} = \frac{N_1}{N_2} = \frac{\theta_2}{\theta_1}$$

$$T_M = J_1\frac{d^2\theta_1}{dt^2} + B_1\frac{d\theta_1}{dt} + T_1 \qquad \text{—— (1)}$$

$$\frac{T_1}{T_2} = \frac{N_1}{N_2}$$

$$T_2 = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L \quad \text{—— (2)}$$

$$T_2 = \left(\frac{N_2}{N_1}\right) T_1$$

$$T_1 \left(\frac{N_2}{N_1}\right) = J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L$$

$$T_1 = \left(\frac{N_1}{N_2}\right)\left[J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L\right]$$

$$T_m = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right)\left[J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L\right]$$

Eliminate θ_2, by putting

$$\theta_2 = \frac{N_1}{N_2}\theta_1$$

$$T_m = J_1 \frac{d^2\theta_1}{dt^2} + B_1 \frac{d\theta_1}{dt} + \left(\frac{N_1}{N_2}\right)\left[J_2 \frac{d^2\left(\frac{N_1}{N_2}\theta_1\right)}{dt^2} + B_2 \frac{d\left(\frac{N_1}{N_2}\theta_1\right)}{dt} + T_L\right]$$

$$= \left[J_1 + \left(\frac{N_1}{N_2}\right)^2 J_2\right]\frac{d^2\theta_1}{dt^2} + \left[B_1 + \left(\frac{N_1}{N_2}\right)^2 B_2\right]\frac{d\theta_1}{dt} + T_L \times \frac{N_1}{N_2} \quad \text{—— (3)}$$

$$T_m = J_1 \frac{d^2}{dt^2}\left(\frac{N_2}{N_1}\right)\theta_2 + B_1 \frac{d}{dt}\left(\frac{N_2}{N_1}\right)\theta_2 + \frac{N_1}{N_2}\left[J_2 \frac{d^2\theta_2}{dt^2} + B_2 \frac{d\theta_2}{dt} + T_L\right]$$

$$= \left[J_1 \frac{N_2}{N_1} + J_2 \frac{N_1}{N_2}\right]\frac{d^2\theta_2}{dt^2} + \left[B_1 \frac{N_2}{N_1} + B_2 \frac{N_1}{N_2}\right]\frac{d\theta_2}{dt} + \frac{N_1}{N_2} T_L$$

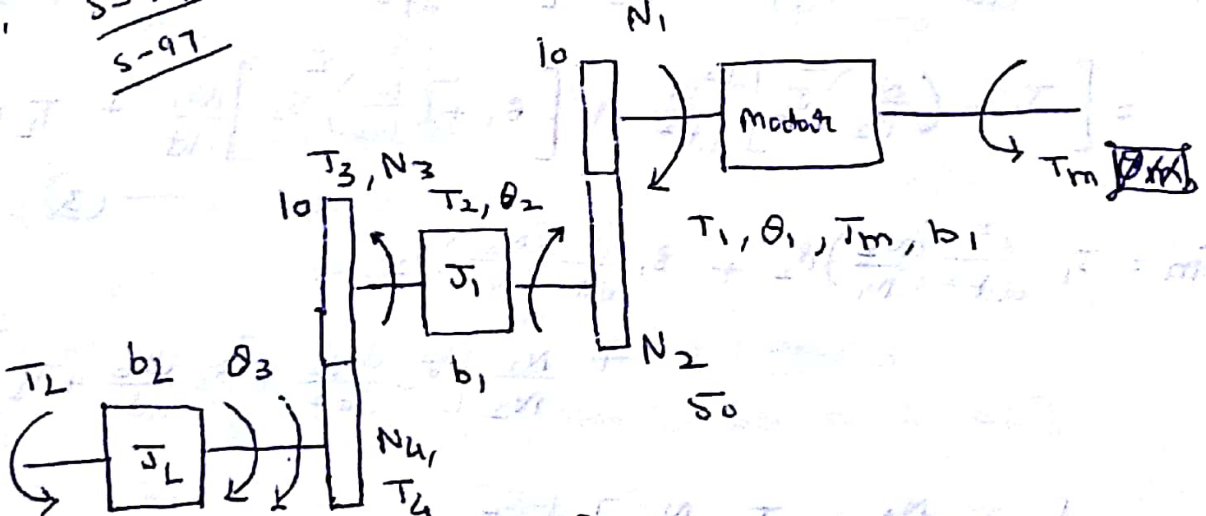
$$T_m \cdot \frac{N_2}{N_1} = \left[J_2 + \left(\frac{N_2}{N_1}\right)^2 J_1\right]\frac{d^2\theta_2}{dt^2} + \left[B_2 + \left(\frac{N_2}{N_1}\right)^2 B_1\right]\frac{d\theta_2}{dt} + T_L \quad \text{—— (4)}$$

Scanned by CamScanner

③ & ④ are the required gear train equations.

90

Ex: S-94
S-97



Derive the T.F for the geared system shown. The no. of teeth are indicated on the gears. The moment of inertia of the load, motor & the intermediate rotor & also the viscous damping coefficient are as given below.

$J_L = 40$ $J_1 = 4$ $J_m = 15\ Kgm^2$ $b_L = 3$ $b_1 = 4$
$b_m = 0.02 \frac{N\text{-}mm}{rad/sec}$

Differential eqn on 1st shaft.

$$T_m = J_m \ddot{\theta}_1 + b_m \dot{\theta}_1 + T_1$$

$$T_m(s) = J_m s^2 \theta_1 + b_m s \theta_1 + T_1 \quad \text{———} \quad (1)$$

on 2nd shaft

$$T_2 = J_1 \ddot{\theta}_2 + b_1 \dot{\theta}_2 + T_3$$

$$T_2(s) = J_1 s^2 \theta_2 + b_1 s \theta_2 + T_3 \quad \text{———} \quad (2)$$

on 3rd shaft

$$T_4 = J_L \ddot{\theta}_3 + b_L \dot{\theta}_3 + T_L$$

$$T_4(s) = J_L s^2 \theta_3 + b_L s \theta_3 + T_L \quad \text{———} \quad (3)$$

$$\therefore \quad T_1 \theta_1 = T_2 \theta_2 \Rightarrow \cancel{T_1 = \left(\frac{T_2}{T_1}\right)}$$

$$T_1 = \left(\frac{\theta_2}{\theta_1}\right) T_2 = \left(\frac{N_1}{N_2}\right) T_2 \quad - (a)$$

also,

$$T_3 \theta_2 = T_4 \theta_3 \Rightarrow T_3 = \left(\frac{\theta_3}{\theta_2}\right) T_4 = \left(\frac{N_3}{N_4}\right) T_4 \quad - (b)$$

$$\theta_2 = \frac{N_1}{N_2} \theta_1 \qquad \theta_3 = \frac{N_3}{N_4} \theta_2 = \frac{N_3}{N_4} \times \frac{N_1}{N_2} \theta_1 \quad - (c)$$

$$\left.\begin{aligned} \theta_2 &= \frac{N_4}{N_3} \theta_3 \\ \theta_1 &= \left(\frac{T_2}{T_1}\right) \theta_2 = \frac{N_2}{N_1} \frac{N_4}{N_3} \theta_3 \end{aligned}\right\} \quad \text{———} \quad (d)$$

$$T_m = J_m s^2 \theta_1 + b_m s \theta_1 + \frac{N_1}{N_2} T_2$$

$$= (J_m s^2 + b_m s)\theta_1 + \frac{N_1}{N_2}\left(J_1 s^2 \theta_2 + b_1 s \theta_2 + \frac{N_3}{N_4} T_4\right)$$

$$= [J_m s^2 + b_m s]\theta_1 + \frac{N_1}{N_2}[J_1 s^2 \theta_2 + b_1 s \theta_2] + \frac{N_1}{N_2}\frac{N_3}{N_4} T_4$$

$$= [J_m s^2 + b_m s]\theta_1 + \frac{N_1}{N_2}[J_1 s^2 + b_1 s]\theta_2 + \frac{N_1}{N_2}\frac{N_3}{N_4}[J_L s^2 + b_L s]\theta_3$$

$$+ \frac{N_1}{N_2}\frac{N_3}{N_4} T_L \quad \text{—— (4)}$$

$$= [J_m s^2 + b_m s]\theta_1 + \frac{N_1}{N_2}[J_1 s^2 + b_1 s]\frac{N_1}{N_2}\theta_1$$

$$+ \frac{N_1}{N_2}\frac{N_3}{N_4}[J_L s^2 + b_L s]\frac{N_1}{N_2}\frac{N_3}{N_4}\theta_1 + \frac{N_1}{N_2}\frac{N_3}{N_4} T_L$$

$$= \left[J_m + J_1\left(\frac{N_1}{N_2}\right)^2 + J_L\left(\frac{N_1}{N_2}\right)^2\left(\frac{N_3}{N_4}\right)^2\right]s^2\theta_1$$

$$+ \left[b_m + b_1\left(\frac{N_1}{N_2}\right)^2 + b_L\left(\frac{N_1}{N_2}\right)^2\left(\frac{N_3}{N_4}\right)^2\right]s\theta_1$$

$$+ \frac{N_1}{N_2}\frac{N_3}{N_4} T_L$$

$$T_m(s) = J_{eq} s^2 \theta_1(s) + b_{eq} s \theta_1(s) + n_1 T_L(s)$$

—— (5)

$J_L = 40 \quad J_1 = 4 \quad J_m = 1.5\ \text{Kg m}^2 \quad b_m = 0.02\ \frac{\text{N-m}}{\text{rad/sec}}$

4 $\Rightarrow$

$$T_m = \left[J_m s^2 + b_m s\right]\frac{N_2}{N_1}\frac{N_4}{N_3}\theta_3 + \left[J_1 s^2 + b_1 s\right]\frac{N_1}{N_2}\frac{N_4}{N_3}\theta_3$$

$$+ \left[J_L s^2 + b_L s\right]\frac{N_1}{N_2}\frac{N_3}{N_4}\theta_3$$

$$+ \frac{N_1}{N_2}\frac{N_3}{N_4}T_L$$

$$= \left[J_m \frac{N_2}{N_1}\frac{N_4}{N_3} + J_1 \frac{N_1}{N_2}\frac{N_4}{N_3} + J_L \frac{N_1}{N_2}\frac{N_3}{N_4}\right] s^2 \theta_3$$

$$+ \left[b_m \frac{N_2}{N_1}\frac{N_4}{N_3} + b_1 \frac{N_1}{N_2}\frac{N_4}{N_3} + b_L \frac{N_1 N_3}{N_2 N_4}\right] s\theta_3 + \frac{N_1}{N_2}\frac{N_3}{N_4}T_L$$

Multiply the above eq^n by $\frac{N_2}{N_1}\frac{N_4}{N_3}$

$$T_m \frac{N_2}{N_1}\frac{N_4}{N_3} = \left[J_L + J_1\left(\frac{N_4}{N_3}\right)^2 + J_m\left(\frac{N_2}{N_1}\right)^2\left(\frac{N_4}{N_3}\right)^2\right] s^2\theta_3$$

$$+ \left[b_L + b_1\left(\frac{N_4}{N_3}\right)^2 + b_m\left(\frac{N_2}{N_1}\right)^2\left(\frac{N_4}{N_3}\right)^2\right] s\theta_3$$

$$+ T_L$$

$$n_2 T_m(s) = J_{eq} s^2 \theta_3(s) + b_{eq} s\theta_3(s) + T_L \qquad \text{—— (6)}$$

$$n_2 T_m(s) - T_L = \left[J_{eq} s^2 + b_{eq} s\right]\theta_3(s)$$

$$\cancel{n_2 T_m}$$

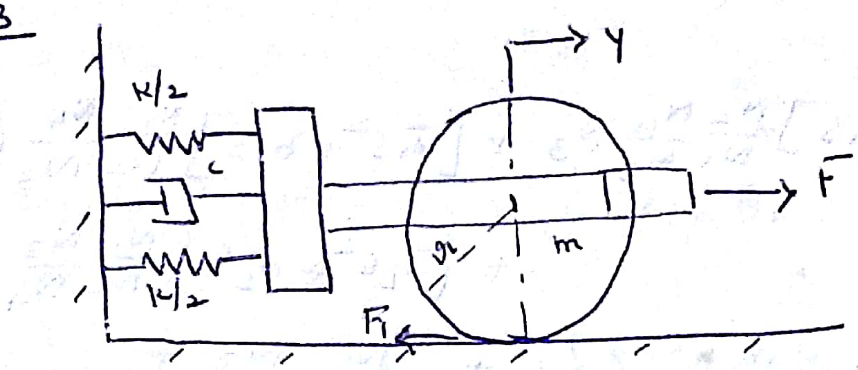
$$\cancel{n_2 - \frac{T_L(s)}{T_m(s)} = \left[J_{eq} s^2 + b_{eq} s\right]\frac{\theta_3(s)}{T_m(s)}}$$

$$n_2 T_m(s)\left[1 - \frac{T_L(s)}{n_2 T_m(s)}\right] = \left[J_{eq} s^2 + b_{eq} s\right]\theta_3(s)$$

$$\frac{\theta_3(s)}{T_m(s)} = \frac{n_2 - \frac{T_L(s)}{T_m(s)}}{J_{eq} s^2 + b_{eq} s}$$

W-93

Ex!



Cylinder rolls without slipping

$K_1 = 100\ N/m$ $\quad m = 24kg$ $\quad \frac{Y(s)}{F(s)} = ?$ $\quad \omega_n = ?$

$$\tan\theta = \frac{y}{r}$$

$$\theta = \frac{y}{r}$$

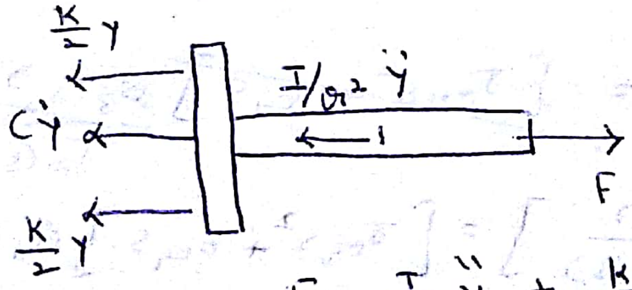
$$y = r\theta$$

$$\dot{y} = r\dot{\theta} \qquad \ddot{y} = r\ddot{\theta}$$

$$T = I\ddot{\theta} = I\frac{\ddot{y}}{r}$$

$$F_1 r = T$$

$$F_1 = \frac{T}{r} = \frac{I}{r^2}\ddot{y}$$



$$F = \frac{I}{r^2}\ddot{y} + \frac{K}{2}y + c\dot{y} + \frac{K}{2}y \quad - (1)$$

$$F(s) = \left[I/r^2\, s^2 + cs + K \right] Y(s)$$

$$\frac{Y(s)}{F(s)} = \frac{1}{\frac{m}{2}s^2 + cs + k}$$ (16) where $I = \frac{mr^2}{2}$

from (i)

$$F = \frac{m}{2}\ddot{y} + c\dot{y} + ky$$

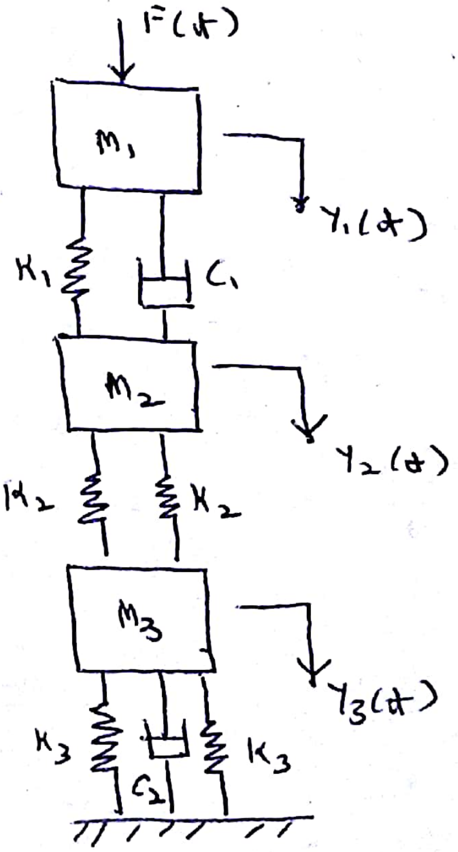
$$\omega_n = \sqrt{\frac{K}{m}} = \sqrt{\frac{K}{m/2}}$$

$$= \sqrt{\frac{100}{24/2}} \qquad \omega_n = 2.8867 \text{ rad/sec}$$

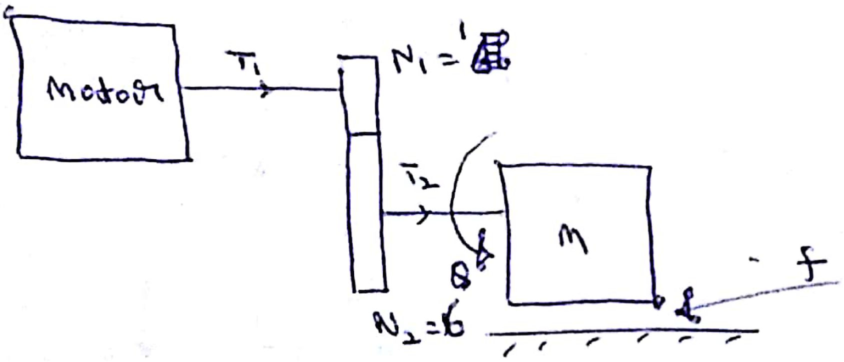
(Ans.)

W-2000

Ex: $\frac{Y_2(s)}{F(s)} = ?$



W-97 Ex: An electric motor produces torque proportional to applied voltage. The motor drives the load thr. speed reduction gear box 6:1 ratio. The load is inertial & viscous type. When a step voltage of 50 volts is applied, the load reaches a speed of 2 rad/sec within 0.5 second. The steady state speed of the load is 3 rad/sec. Find the T.F. relating motor angular rotation to the applied voltage. Assume that motor generates 0.1 N-m of torque/volt. Neglect Inertia of Motor shaft.



$$\frac{T_1}{T_2} = \frac{N_1}{N_2}$$

$$T_2 N_1 = T_1 N_2$$

$$T_2 = \frac{N_2}{N_1} T_1$$

$$T_2 = \frac{6}{1} T_1$$

$$N_1 T_1 = N_2 T_2$$

$$T_2 = \frac{N_1}{N_2} T_1 \;/\; 6 \times T_1 = 6 T_1$$

$$T_2 = J\ddot{\theta} + f\dot{\theta}$$

$$T_2 = J\ddot{\theta} + f\dot{\theta} = J\dot{\omega} + f\omega$$

$$T_2(s) = (Js + f)\,\omega(s)$$

$V \propto T_1 \quad V = kT_1 \quad V = 10T_1 \quad \left[\because \frac{V}{T_1} = \frac{1}{0.1}\right]$

$$T_2 = 6 \times \frac{V}{10} \Rightarrow T_2 = 0.6V$$

$$T_2(s) = 0.6V(s)$$

$$0.6V(s) = (Js + f)\,\omega(s)$$

$$\frac{\omega(s)}{V(s)} = \frac{0.6}{Js + f} = \frac{0.6/f}{\frac{J}{f}s + 1} = \frac{K_1}{\tau s + 1}$$

$$\& \quad \tau = \frac{J}{f} \qquad \text{where } K_1 = \frac{0.6}{f}$$

Scanned by CamScanner

Replace "s" by "D"

$$\frac{\omega(s)}{V(s)} = \frac{K_1}{\tau D+1}$$

$$\left(\frac{J}{f}D+1\right)\omega = \frac{0.6}{f}V$$

Comparing the above eq^n with.

$$(\tau D+1)\theta_o = K\theta_i$$

& comparing with sol^n

$$\theta_o = K\theta_i(1-e^{-t/\tau})$$

$$\omega = \frac{0.6}{f}V\left(1-e^{-t/J/f}\right) \quad \text{——} \quad (1)$$

According to the 1st condition $t = 0.5\,sec$ $\omega = 2\,rad/sec$ $V = 50\,Volts$

$$2 = \frac{0.6}{f}\times 50\left(1-e^{-t/J/f}\right) \quad \text{——} \quad (a)$$

According to the next cond^n

At steady state $t-\infty$

then eq^n (1) reduces to

$$\omega = \frac{0.6}{f}\times V(1-0)$$

$$\omega = \frac{0.6}{f}\times V \quad \text{——} \quad (b)$$

$$3 = \frac{0.6}{f}\times 50$$

$$f = 10\,\frac{N-m}{rad/sec}$$

$t \div \frac{J}{f}$

$T = f\frac{d\theta}{dt}$

$f = \frac{N-m}{\frac{rad}{sec}}$

$$2 = \frac{0.6}{10}\times 50\left(1-e^{-\frac{t\times 10}{J}}\right)$$

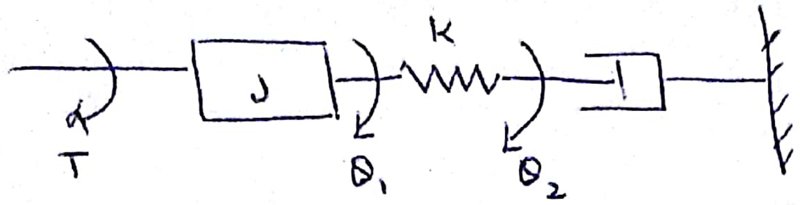
$$\frac{2}{3} = 1 - e^{-\frac{t\times 10}{J}}$$

$$\frac{2}{3} = 1 - e^{-\frac{0.5\times 10}{J}}$$

$$-\frac{0.5\times 10}{J} = \ln\frac{1}{3} \qquad J = 4.5512$$

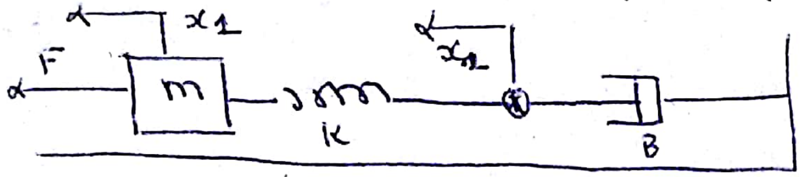
$$\boxed{\frac{\omega(s)}{V(s)} = \frac{0.6/10}{\frac{4.5512}{10}s+1} = \frac{0.06}{0.45512s+1}}$$

Ex:



$$\frac{\theta_1}{T} = ?$$

Similar translational system:



$$I\alpha = \Sigma T$$

$$T = J\ddot{\theta}_1 + K(\theta_1 - \theta_2)$$

$$J\ddot{\theta}_1 = T - K(\theta_1 - \theta_2)$$

$$K(\theta_1 - \theta_2) = B\dot{\theta}_2$$

$$Js^2\theta_1 = T - K(\theta_1 - \theta_2)$$

$$Bs\theta_2 = K(\theta_1 - \theta_2) \qquad [Bs + K]\theta_2 = K\theta_1$$

$$\theta_2 = \frac{K(\theta_1 - \theta_2)}{Bs} \qquad \theta_2 = \frac{K\theta_1}{K + Bs}$$

$$Js^2\theta_1 = T - K\left[\theta_1 - \frac{K(\theta_1 - \theta_2)}{Bs}\right]$$

$$Js^2\theta_1 = T - K\left[\theta_1 - \frac{K\left(\theta_1 - \frac{K\theta_1}{K + Bs}\right)}{Bs}\right]$$

$$BsJs^2\theta_1 = TBs - K\theta_1\cancel{B}Bs + K^2\left[\theta_1 - \frac{K\theta_1}{K + Bs}\right]$$

$$\theta_1\left[BsJs^2 + KBs - K^2 + \frac{K^3}{K + Bs}\right] = TBs$$

$$\frac{\theta_1}{T} = \frac{Bs}{BsJs^2 + KBs - K^2 + \frac{K^3}{K + Bs}}$$